1. Complexity, scientific creativity and clustering

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INTRODUCTION

By the late 1960s, economists and policymakers realized that manufacturing employment was stagnating in many of the most developed countries. In a pioneering study of the American economy, Fritz Machlup (1980) concludes that what he calls the "knowledge industry" already accounted for close to 30 percent of America's gross domestic product (GDP) in the 1970s. Using a definition of knowledge-producing occupations that included both occupations assumed to produce new knowledge and those involved in transferring knowledge and information, Machlup also showed that this group of occupations had grown faster than other groups for several decades. Åke Andersson (1985) provides a broad historical overview of creativity, infrastructural change and economic growth, suggesting that an assessment of the relative knowledge orientation of an economy should exclude occupations that focus on information transmission. Andersson's definition of "knowledge-handling occupations" is thus a subset of Machlup's knowledge-producing occupations, and mainly consists of workers in education, research and the arts. Andersson and Persson (1993) show that the number of workers in this occupational category is growing faster than other categories in Sweden and other advanced economies.

In a more detailed study of different industries, Christer Anderstig and Björn Hårsman (1986) show that the percentage of "knowledge-handlers" is increasing in manufacturing as well as in services; they further show that this expansion is fastest in large metropolitan areas. Edward Glaeser and David Mare (2001) and Richard Florida (2002) elaborate upon these early findings in an American context, with the conclusion that U.S. metropolitan areas increasingly depend on highly educated workers or a new "creative class."

In the 1970s and 1980s, producers in the electronic, pharmaceutical, optical and transport industries were becoming increasingly aware of the need for research and development (R&D) in order to secure dynamic comparative advantages in their location and trading decisions. Public policy, however, lagged behind. It was only later that actors in the public sector realized that there was a need for a corresponding shift in their education and science policies. The European Institute of Innovation and Technology, EIT, a body of the European Union, can be looked upon as a political manifestation of a perceived need for increased collaboration between universities and industry to enhance the competitiveness of Europe. Through long-term and large-scale investments in "knowledge and innovation communities," EIT aims at fostering the innovation capacity of Europe. The communities or so-called "KICs" are run and co-funded by university and industry stakeholders, integrating higher education, research and business. By way of example, the EIT ICT Labs include universities in Berlin, Eindhoven, Helsinki, Paris, Stockholm and Trento and firms such as Ericsson, Siemens, Philips and Nokia. The budget for the EIT ICT Labs amounts to about €70 million per year. Figure 1.1 is an attempt to visualize the relationships among the different aspects of knowledge, research and production (Andersson and Beckmann, 2009).

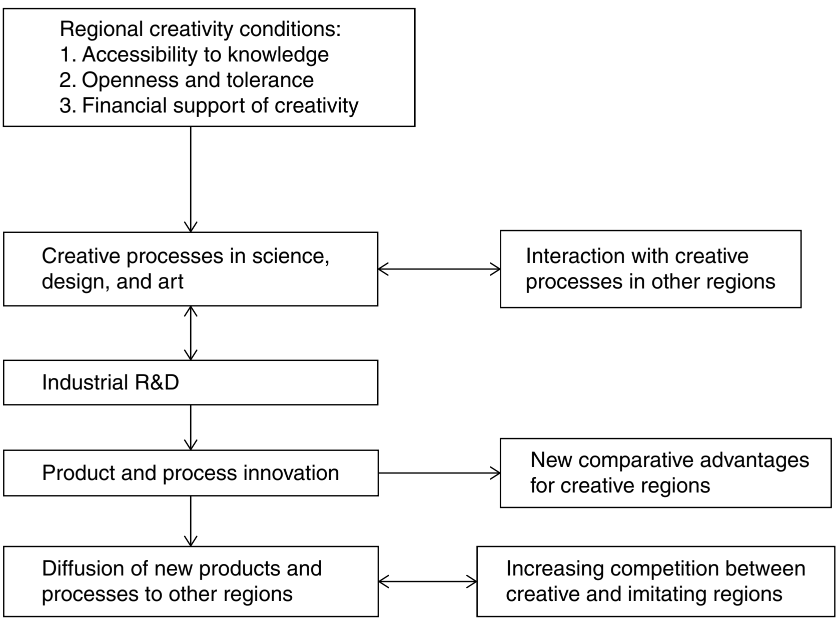

Figure 1.1 Creativity, research and new comparative advantages

Creativity in research is nowadays a precondition for industrial R&D, in contrast to the situation in earlier stages of development. In early industrial society, there were clear "stages of innovative production" whereby the lone inventor (often with some short engineering education) provided the industrial entrepreneur with the key input for a successful innovation. Science was for the most part external to this production process. Even today the managers of some high-technology firms have tried to cut off the links to science by planning the whole process from scientific inputs to innovative outputs. It is likely that such strategies misallocate knowledge resources. The increasing complexity of products and production systems necessitates a stronger interaction between the creativity of basic science and the technological R&D of firms.

Our contention is that scientific creativity is becoming increasingly important for industrial research and innovation. This will not only affect the location of industries and their knowledge capital. It will also change the location of scientists. A study by Hans Lööf and Pardis Navabi (2013) quantifies the benefits of knowledge-intensive locations. They analyse 10,000 Swedish export firms over a twelve-year period in order to estimate the relationship between productivity and innovations. Their conclusion is that both persistent and non-persistent innovators benefit from a knowledge-intensive local environment of universities and other scientific research institutions. Additionally, this type of external knowledge influences the productivity growth of those firms that are classified as persistent innovators.

FROM SCIENTIFIC CREATIVITY TO INDUSTRIAL R&D

Fundamental creativity in the sciences is shaping the future comparative advantages of industry. The intermediate factor is industrial R&D, with product and process innovations as direct outputs. Still, scientific research accounts for only a modest part of overall spending, even in those economies that are most "science-intensive." Spending on industrial R&D—which tends to be more incremental and short-sighted than scientific research—is more than five times greater than spending on scientific research in the Organisation for Economic Co-operation and Development (OECD) countries (see Table 1.1).

As Table 1.1 shows, the public sector—which is the main source of research funding in the higher education sector—attaches less importance to basic research than the private sector attaches to applied industrial research (or, in some cases, than the public sector attaches to applied military research). There are at least two reasons for this.

Table 1.1 Research and development spending in the OECD countries, 2008–2011

Country	Overall R&D (percent of GDP)	Scientific research (percent of GDP)
Sweden	3.43	.90
Denmark	3.06	.90
Finland	3.89	.79
Netherlands	1.83	.75
Switzerland	2.97	.72
Austria	2.71	.72
Canada	1.80	.69
Iceland	2.64	.66
Estonia	1.62	.62
Portugal	1.59	.59
Norway	1.69	.55
Australia	2.18	.54
Germany	2.79	.51
Ireland	1.79	.51
France	2.12	.48
United Kingdom	1.70	.48
Belgium	1.97	.46
Japan	3.33	.45
OECD	2.17	.44
New Zealand	1.30	.43
South Korea	3.58	.40
United States	2.28	.39
Spain	1.34	.39
Turkey	.84	.39
Italy	1.25	.36
Greece	.60	.30
Slovenia	1.85	.29
Czech Republic	1.56	.28
Poland	.74	.27
Luxembourg	1.56	.19
Slovakia	.62	.17
Hungary	.23	.16
Chile	.39	.16
Mexico	.37	.10

Source: OECD (2012).

First, scientific research tends to adopt a much longer time perspective than industrial R&D. There is consequently greater uncertainty of success in the sense of Ramsey (1927, pp. 153–170; see also Sahlin, 1990, pp. 88–94). Second, science focuses less on financial or private returns and this characteristic induces an evaluation problem among policymakers. Third, scientific returns are in general public in their effects, as a consequence of scientists' preference for open-access dissemination of their results.

Politicians and bureaucrats often assume that heavily marketed "national innovation policies" are best at promoting future comparative advantages. The implication is that they tend to support industrial R&D rather than scientific research, in spite of the (potentially) much greater and more widespread social returns that scientific creativity brings about.

William Baumol (2004) suggests that policymakers should distinguish between inventors' inventions, entrepreneurial innovations and firms' R&D-based inventions and innovations. Though corporations may employ large numbers of scientists and operate research centres in different countries, independent inventors and firm-external entrepreneurs provide disproportionate numbers of radical—as opposed to incremental—innovations. Baumol (ibid., p. 8) writes that

> the R&D divisions of the large firm tend primarily to require personnel who have undergone training for mastery of extant information and analytic methods, while the work of the independent entrepreneur and inventor may prove to be more effectively facilitated by avoidance of that sort of preparation to the extent that it impedes imagination and originality.

A counter-argument to Baumol's hypothesis is that increasing complexity may lead to the demise of the classical independent inventor, working alone in some minimally equipped laboratory. As Rogers Hollingsworth (2007) points out, the increasing complexity of fields such as biomedicine means that large and multidisciplinary science teams are becoming increasingly important for basic scientific research, which is the domain that has always been responsible for the most revolutionary creative breakthroughs.

Economies and their research organizations are becoming increasingly complex. We therefore need an understanding of what the increasing complexity of products and processes imply for the organization of economic life. There are numerous proposed definitions of complexity. Most of these are somewhat intuitive. Ray Solomonoff (1964), Andrey Kolmogorov (1965) and Gregory Chaitin (1966) provide a mathematical and precise definition of complexity. They claim that complexity is measurable and defined as the minimal length of a program or algorithm that yields an exact solution to a pre-formulated problem. This can be clarified with a few simple examples. An illustrative problem is to find the shortest way of generating a given sequence of numbers.

Example 1: 1010101010. . .
Example 2: 00200030000400000500000006. . .
Example 3: 12154369982134579870981269994333.

It is easy to identify formulae or algorithms that are shorter than the series of Examples 1 and 2 and that yield the same unique number series. Both of these formulae of minimal length are short and thus of low complexity. In contrast, no formula shorter than the series itself can identify Example 3, which is a random series of numbers. Example 3 is thus more complex than Examples 1 or 2. Example 2 is, however, more complex than Example 1.

It is possible to generalize the complexity of mathematical expressions and computer algorithms to phenomena such as blueprints and production process instructions (Casti and DePauli, 2000; Solomonoff, 1964). Standard goods must follow strict and stepwise (i.e. algorithmic) rules of composition. A simple example is a baker's recipe. For example, the recipe for a standard American brownie is less complex than the recipe for the *Sachertorte* of the Hotel Sacher in Vienna (we believe rather than know this, since this particular *Sachertorte* recipe is a trade secret). But despite the clarity of this example, it also alludes to numerous limitations of the computational complexity measure.

The first limitation arises from the difference between numbers in an algorithm and ingredients in a cake. Cake ingredients have much greater *scope* than numbers; they are heterogeneous rather than uniform in having an open-ended set of underlying attributes.

A second limitation is that cakes, unlike numbers, are sensitive to the skills of the individual using the recipe. A recipe-using individual is not as homogeneous as an algorithm-using computer. A skilful worker can adjust the recipe if the delivery of an ingredient is delayed or if an ingredient is of superior or inferior quality (in one or more of its intrinsic attributes). The structure of inputs in the production process is thus a function of algorithmic complexity, c_A.

Baumol (2004, p. 8) captures the skill change that accompanies changes in algorithmic complexity when he writes that

> incremental improvement of complex products may require mastery of far more demanding technical information and techniques than was needed for the original ideas that resulted in the invention of those products. The technology needed to improve the design of a Boeing-777 passenger airplane is obviously enormously more complex than that underlying the Wright brothers' first vehicle.

The *cost* of production will thus depend on (at least) three factors: the complexity of the recipe (composition or design); the quality attributes of the ingredients (land and physical capital); and the skills of the worker (labour or human capital).

Besides the complexity of production there is also the complexity of consumption. These two types of economic complexity do not coincide. More complex production often causes *less* complex consumption. One example is computer-assisted automatic transmission systems in cars. The production complexity is substantial, but the user derives utility from this *high* production complexity to the extent that s/he values *low* consumption complexity, c_{IJ}. This implies that revenues as well as costs depend on both production *and* consumption complexity.

We can assume that consumption complexity is a function of production complexity, as specified above. This means that we can now formulate a profit function, Y, for a given good as follows:

$$Y = R(\mathbf{q}, c_U(c_A, \mathbf{v})) - C(\mathbf{q}, \mathbf{v}, c_A); \tag{1.1}$$

where
$\mathbf{v}$ = a vector of ordinary inputs (dependent on the prevailing complexity levels);
$\mathbf{q}$ = vector of scales of production of the given set of products.

In the short term, we treat the complexities of the products as given by earlier investments in new knowledge. In the long term, algorithmic complexity, input structure and skills can all change as a result of creativity in science and in applied research. Such changes are possible only on much slower timescales than the typical timescale of ordinary business decisions, which in this case includes R&D-induced incremental improvements.

The accumulation of scientific knowledge thus occurs through a slow and creative process that changes the algorithmic complexity of a given good:

$$\begin{aligned} \Delta C/\Delta\tau &= F(C, S_r) \\ \Delta c_A/\Delta\tau &= G(c_A, C); \end{aligned} \tag{1.2}$$

where
C = scientific creative capital;
S_r = funds allocated to scientific research;
τ = a basic long time period.

We assume the functions to be concave with respect to C, S_r and c_A, ranging from R^+ to R^+. In addition, τ = kt, where k is a constant transforming ordinary time (say years) into the much longer time periods that new scientific breakthroughs usually need in order to mature and gain acceptance in the community of scientists. By adiabatic approximation $(G, F) \approx 0$, implying that firms will treat the current stock of established scientific knowledge as a fixed constraint on their opportunity set. We may therefore treat the algorithmic complexity of a specific long time period as the knowledge infrastructure. Firms treat this infrastructure as a stable basic input into the much more rapid industrially applied and incremental research processes that transform scientific results into product and process innovations.

CREATIVITY AND COMPLEXITY

There are numerous contemporary examples of how the creativity of scientists, designers and engineers has increased product complexity and necessarily *preceded* industrial innovations. The most important example is the information and communications technology (ICT) industry. One should keep in mind that recent rapid increases in the product complexity of computers and ICT systems would not have been possible without the scientific creativity of Alan Turing and John von Neumann in the 1930s and 1940s.

The transport equipment industry is another example of this process of increasing interdependency between science and industrial technology. A modern airplane is more energy-efficient and easier to fly than vintage airplanes from the 1920s. But it was only through a massive and protracted accumulation of science-based novel electronic and hydro-mechanical components that this combination of high production and low consumption complexity was made possible. It is also noteworthy that workers in contemporary airplane manufacturing plants have more advanced technical skills than their counterparts in the 1920s.

The trend towards increasing levels of complexity, which is now evident in most industries, eventually caused a restructuring of both scientific and industrial research. It also gave rise to the spread of higher education and the current popularity of national and regional innovation policies.

Although the Second World War accelerated the integration of scientific creativity and product development in the military, a more wide-ranging civilian integration only began in the 1970s. Two industries were then at the forefront, integrating science, industrial research and technological innovation.

The computer industry is one of these industries. It emerged from the rather simple office equipment industry. The British mathematician Alan Turing supplied the key scientific input. Turing had been analysing the mathematical properties of digital mathematics in a simple but very general algorithm, which was later to be known as Universal Turing Machines. Basing their work on Turing's theorems, two Princeton mathematicians, John von Neumann and Herman Goldstine, provided the original inventive input in the 1950s: the first prototype of the programmable computer. It was only later—and sometimes with great resistance—that industry became engaged in the development of programmable computers.

The increase in the product complexity of computers since the 1950s has been staggering. Algorithmic complexity has been increasing at an unrivalled pace from the 1950s onward. It was not only Turing, Neumann and Goldstine who made the resulting "information revolution" possible. Their scientific predecessors also contributed to the necessary conditions of this particular scientific breakthrough. Examples include Sir Isaac Newton (the Newton–Raphson technique for solving non-linear equations), George Dantzig (the simplex method for solving linear programming problems) and other algorithmically oriented mathematicians.

Information technology is but one example of this type of process that connects scientific breakthroughs with increasingly complex products. A non-exhaustive catalogue of creative activities in sciences which have caused increasing downstream product complexity would include aerospace engineering, architecture, astrophysics, biochemistry, chaos theory, meteorology, electrical engineering, financial engineering, geophysics, integer optimization, molecular biology, pattern recognition, physics, robotics, systems engineering and transport engineering.

The algorithmic complexity of computer programs has relied on the increasing hardware capacity of computers as well as on the need for decreasing consumption complexity. The user-friendly laptop computer is one outcome of this parallel development. Perhaps the most important aspect of the dynamics of hardware complexity is ever-increasing storage capacity. IBM introduced the 350 Disk File in 1956. This hard drive had a total capacity of only five million characters. In 2011 Hitachi introduced the first four-terabyte hard drive, and in 2012 Victorinox began selling one-terabyte USB flash drives (one terabyte or TB corresponds to one trillion bytes or eight trillion single characters of computer text).

Greater speed has accompanied greater storage capacity. A research team at the University of California, San Diego has for example developed a "phase-change memory solid state storage device," which is thousands of times faster than the fastest conventional hard drives. Another team at Johns Hopkins University has discovered some previously unknown properties of a common memory material (GST), paving the way for the development of novel memory drives that store and retrieve data more quickly, as well as being more durable. These examples illustrate how scientific breakthroughs cause new inventions, and how subsequent product innovations depend on interactions between parallel increases in algorithmic complexity, (physical) input complexity and skill complexity.

Another example of an industry which had an early start in the integration of scientific and industrial creativity is the pharmaceutical industry. Decision-makers in that industry early on identified and acted on the need for an integration of medical, chemical and biological science with firms' research and innovation strategies. Yet another example is the medical equipment industry, whose constituent firms have developed elaborate strategies for using relevant results from physics, computer science and cognitive science.

This raises the question of what type of organization is best able to cultivate the scientific breakthroughs that firms subsequently use in their product development efforts. Hollingsworth (2007) addresses this question by connecting scientific complexity to the frequency of creative breakthroughs and the internal organization of universities, research institutes and laboratories. His focus is on biomedical science, which is unusually concerned with understanding and predicting highly complex systems. Hollingsworth (ibid., p. 129) notes that "high cognitive complexity is the capacity to observe and understand in novel ways the relationships among complex phenomena, the capacity to see relationships among disparate fields of knowledge. And it is that capacity which greatly increases the potential for making a major discovery."

Hollingsworth's empirical analysis distinguishes between two different types of laboratories (A and B). He starts by asking (ibid., p. 131) the following question: "What were the characteristics of the culture and the structure of the laboratory where the [breakthrough] research occurred?" The initial finding was that most labs never made any major discoveries, including illustrious ones such as the Royal Society or the National Academy of Sciences. Meanwhile, there was a small subset that did in fact produce revolutionary scientific results. Hollingsworth labels the creative type of lab "Type A," while he calls the relatively uncreative lab "Type B." As it turns out, labs which produced creative breakthroughs had similar Type A organizational attributes, although most Type A labs were unsuccessful. More significantly, *none* of the conventionally structured Type B labs ever produced a truly revolutionary biomedical finding. Table 1.2 illustrates the organizational attributes of Type A and Type B labs.

Hollingsworth's results are striking. He writes (ibid., p. 132) that "almost all of the 291 discoveries in our project were made in Type A laboratories . . . Type B laboratories are at the opposite end of the continuum on virtually all the lab characteristics. Significantly, none of the 291 discoveries in our research occurred in Type B labs." Hollingsworth's

Table 1.2 Organizational attributes of Type A and Type B laboratories

Attribute	Type A lab	Type B lab
Cognitive	High scientific diversity	Low or moderate scientific diversity
Social	High and diversified network connectivity	High network connectivity within a single discipline
Material	Access to funding for high-risk research	Limited funding for high-risk research
Personality of the lab leader	High cognitive complexity; high confidence; high motivation	Low cognitive complexity; risk-averse
Leadership	Excellent grasp of how different fields may be integrated	Not concerned with integrating distinct scientific disciplines

Source: Hollingsworth (2007, 2009).

findings imply that one may doubt the wisdom of the current tendency towards increased specialization in the sciences, which has been the normal outcome of the interplay of relevant forces such as monopolistic competition among scientists, the funding policies of science foundations and universities' hiring strategies.

On the basis of these results, we conclude that the development of science towards increasingly complex theories, models and products causes a need for more complex cognitive capacity among individual creative scientists as well as within laboratories and other research organizations at the micro level. Moreover, the increase in expected (but uncertain) individual creativity that accompanies good access to new and diversified knowledge (cf. Andersson, 1985; Florida, 2002; Hollingsworth, 2007; Simonton, 2011) provides strong arguments for locating such micro research organizations in large, open and diverse cities.

THE GLOBAL AND REGIONAL DISTRIBUTION OF CREATIVE SCIENTIFIC CAPACITY

The most systematic studies of the global distribution of scientific creative capacity are all based on Thomson Reuter's Science Citation Index, which records publishing and citations in peer-reviewed scientific journals.

Table 1.3 Science output in Europe's 25 leading regions, 2008–2010

Rank	Labour market area	Country	SCI publications
1	London	United Kingdom	96,856
2	Paris	France	77,007
3	Randstad (Amsterdam–Rotterdam–Utrecht)	Netherlands	65,527
4	Moskva (Moscow)	Russia	45,857
5	Madrid	Spain	41,926
6	Berlin	Germany	41,923
7	Øresund (København–Malmö–Lund)	Denmark–Sweden	38,970
8	Milano (Milan)	Italy	37,917
9	Roma (Rome)	Italy	37,681
10	Barcelona	Spain	36,657
11	Stockholm–Uppsala	Sweden	35,257
12	München (Munich)	Germany	32,132
13	Oxford–Reading	United Kingdom	30,374
14	Manchester–Liverpool	United Kingdom	30,144
15	Dortmund–Düsseldorf–Köln	Germany	29,351
16	Edinburgh–Glasgow	United Kingdom	29,151
17	Cambridge	United Kingdom	26,927
18	Frankfurt	Germany	26,221
19	Genève–Lausanne	Switzerland	25,996
20	Zürich	Switzerland	25,720
21	Wien (Vienna)	Austria	24,084
22	Mannheim–Heidelberg	Germany	23,324
23	Sheffield–Leeds	United Kingdom	22,306
24	Basel–Mulhouse–Freiburg	Switzerland–France–Germany	22,295
25	Bruxelles–Antwerpen (Brussels–Antwerp)	Belgium	21,957

Source: Thomson-Reuters (2012).

Admittedly, only a very low percentage of published papers are truly creative. Although the world's highest ranked universities according to popular league tables are mostly located in a few American and British regions, the density of research output in some of the smaller European countries such as Denmark, Sweden and Switzerland is quite remarkable. But even in Europe, the distribution of research output is heavily concentrated in large metropolitan areas and a handful of college towns. Table 1.3 gives the total number of publications registered in the Science Citation Index (SCI) from 2008 to 2010 in European urban regions. The regions are delimited according to the spatial extent of a time distance of forty-five minutes from the regional centre by the fastest transport mode.

Historical data (Matthiessen et al., 2011) show that there is considerable stability in the geographic distribution of European science production, with centres of gravity—both in terms of mass and network connectivity—in the three leading nodes of London, Paris and Amsterdam (Randstad).

Denmark and Sweden spent the most money on scientific research relative to GDP, while Switzerland (ahead of Denmark and Sweden) produced the most SCI papers relative to population between 2008 and 2010 (Andersson and Andersson, 2013). As Table 1.3 shows, the Øresund region (which includes Copenhagen in Denmark as well as Malmö and Lund in Sweden) is Europe's seventh largest agglomeration of science production, while Stockholm–Uppsala is in eleventh place. More detailed national statistics show that Sweden's science output is disproportionately located in Stockholm (narrowly defined) and in the two university towns of Lund and Uppsala (Matthiessen et al., 2011). There is also an over-representation of scientists in other Swedish regions with research universities, for example in Göteborg (Gothenburg), Umeå and Luleå. But what *mechanisms* cause scientists to locate in one rather than another region?

ACCESSIBILITY AND CLUSTERING OF SCIENTISTS—A THEORETICAL APPROACH

Assume space to be subdivided into N regions. The scientist must select one location from these N regions. One location preference argument is thus the accessibility to other scientists from a region i to all N areas. Scientists can be expected to prefer other scientists to have proximate rather than distant locations. Similarly, scientists are expected to aim for the greatest possible number of other scientists at a given distance.

The definition of accessibility to scientists from area i is

$$a_i^x = \sum_{j=1}^{N} f(d_{ij})\, x_j \tag{1.3}$$

where

a_i^x = accessibility to scientists, x, from area i;

$f(d_{ij})$ = a strictly decreasing function of the distance between two areas i and j;

x_j = the number of scientists in area j.

The positive convex distance function $f(d_{ij})$ implies that the accessibility to a given number of scientists decreases with increases in the distance between regions i and j.

A reasonable and commonly used measure of $f(d_{ij})$ is

$$e^{-\beta dij} \text{ with } \beta \geq 0 \text{ and } d \geq 0. \tag{1.4}$$

Here d is defined as the time distance by the transport mode that scientists normally use. This functional form implies that accessibility is a spatial analogue to the discounted total value of revenues in capital theory. An advantage of this functional form is the property that $e^{-\beta d}$ has the limits 1 if $d \to 0$ and 0 if $d \to \infty$.

The second important factor in scientists' location choices is the scientific infrastructure in terms of labs, libraries, supercomputers and other large-scale science machinery in universities and R&D-intensive firms. Examples of large-scale infrastructures in science include CERN in Geneva, the new ESS unit in the Øresund region and the IBM research facility in Connecticut.

Scientific infrastructural accessibility can be denoted as $a_i^s = \sum_j^N e^{-cdij} S_j$, where S_j = scientific infrastructure in region j. The valuation of accessibility to such infrastructure would of course decline with increases in the number of users, as a reflection of bottlenecks and other congestion phenomena.

A third key factor in the choice of location is the relative level of science subsidies in region i, W_i. We may further assume that the level of algorithmic complexity, c_A, is fixed in the short run and acts as a vector of level parameters in the **Q**-mapping.

It has been shown that a non-linear Eigenvalue equation determining the endogenous clustering has an equilibrium solution with a positive equalized rate of return ρ on the associated positive human capital vector **x** in:

$$\rho \mathbf{x} = \mathbf{Q}(\mathbf{x}, a^s, \mathbf{W}, c_A); \tag{1.5}$$

where $\mathbf{Q}(\mathbf{x})$ is a continuous, quasi-concave mapping from R^+ to R^+ (Nikaido, 1968). For such a system, we can use one of Nikaido's (1968) theorems as follows.

Assumptions:

a. $\mathbf{Q}(x) = (Q_i(x))$ is defined for all $x \geq 0$.
b. $\mathbf{Q}(x)$ is continuous as a mapping $\mathbf{Q}$: $R_+^n \to R_+^n$ except possibly at $x = 0$.
c. $\mathbf{Q}(x)$ is positively homogenous of order m, $0 \leq m \leq 1$ in the sense that $\mathbf{Q}(x) \geq 0$ and $x \geq 0$.

Theorem:

$$\text{Let } \Lambda = \{\boldsymbol{Q}(x) = \rho x\} \text{ for } x \in p_n \tag{1.6}$$

where

$$p_n = \left\{ x | x \geq 0, \sum_{i=1}^{n} x_i = 1 \right\} \tag{1.7}$$

is the standard simplex. Then $\mathbf{Q}(x)$ contains a maximum characteristic value which is denoted $\rho(\mathbf{Q})$. Furthermore, if $\mathbf{Q}(x)$ is homogenous of degree 1, that is if $m = 1$ as a special case of assumption (c), then $\rho(\mathbf{Q})$ is the largest of all the Eigenvalues of $\mathbf{Q}(x)$.

Proof: Nikaido (1968).

In the vicinity of an equilibrium, $\rho(\mathbf{Q})$ can be locally linearized as $\mathbf{Q}(x) = \mathbf{Q}(x^*)$, where x^* is x in equilibrium. We then have the Eigen-equation

$$\rho \mathbf{z} = \mathbf{Q}(x^*)\mathbf{z} \tag{1.8}$$

with the equilibrium growth rate $\rho(\boldsymbol{Q}(x^*))$. Applying the Frobenius–Perron theorem we can conclude that the rate of return to scientific capital, ρ, will increase with

a. an increase in the scientific infrastructure in any region,
b. a decrease in any distance, d_{ij},
c. a decrease in the general distance friction, β,
d. an increase in the rate of subsidies to scientific research in any region.

Changes of types a–d, as specified above, influence the rate of return to science capital in all nodes to the same extent—but only in the long run after attaining general equilibrium.

CONCLUSIONS

Inventions and other forms of creativity depend on external support in the form of funding. In recent decades this has become a policy priority in government and industry, reflecting a search for dynamic comparative advantages. Nevertheless, the relative share of R&D expenditures allocated to basic scientific research in universities is between 10 and 30 percent in most countries. This share is especially low in East Asian countries such as China, Japan and Korea. One possible cause of the low share allocated to the scientific infrastructure is the myopic nature of political decisions as compared with the necessary foresight of more than a decade in high-tech industries focusing on increasingly complex products and production systems.

In this chapter we propose a new approach to modelling the complexity of products. The approach uses definitions that were originally proposed in another context by Chaitin, Kolmogorov and Solomonoff. The basic idea is that the slow and steady accumulation of scientific research generates increasingly complex theories. Such theories and models are potentially useful for the more rapid industrial R&D of products and production systems. In our model, the decision-makers that shape firms' R&D strategies treat established scientific theories as a fixed knowledge infrastructure.

Hollingsworth has shown empirically that the organization of complex scientific research projects that aim at scientific breakthroughs has to be carried out by teams that are much more interdisciplinary and open-minded than is normal in firms' R&D efforts.

We conclude that the development of science toward increasingly complex theories, models and products causes a need for more complex cognitive capacity among individual scientists as well as within laboratories and other research organizations at the micro level. Moreover, the increase in expected (but uncertain) individual creativity that accompanies good access to new and diversified knowledge provides strong arguments for locating such micro-level research organizations in large, open and diverse city regions.

Hence we expect science to be clustered in space. In line with this expectation we formulate a spatial general equilibrium model of science clusters. This model is a non-linear Eigen-equation, in which scientific collaboration gains, R&D support systems and the level of complexity of products and production systems act as joint determinants of scientists' clustering patterns.

Statistics on the scientific output of European city regions show that the distribution is clustered in a limited number of regions. Twenty of the twenty-five leading regions are located in the north-western quadrant of Europe, alluding to a future super-region of comparative advantages in the production of science and complex products that encompasses leading city regions in Benelux, Britain, northern France, western Germany, Scandinavia and Switzerland. There are few regional analogies in the south or east, and none at all in the south-east.

Some policy conclusions can be formulated. The first and most obvious conclusion is related to national knowledge policy. Scientific research has in contemporary times provided the knowledge infrastructure of industrial R&D investments. The importance of this infrastructure for the productivity of R&D in the industries is growing with the growth of complexity of products and production systems. However, most of the OECD countries spend a very small part of total R&D investments on scientific research. An increase is urgently needed in most countries and especially in Japan and the Asian tiger economies.

A second policy conclusion is related to the growing contradiction between scientific specialization to gain comparative advantages and the need for diversified science laboratories and institutes to handle the increasing complexity, as shown by Hollingsworth. The current focus on publication quantities and citation frequencies by university and research-funding administrations in their selection of scientists to be supported tends to favour scientific specialists, with detrimental consequences for creativity in complex research fields. Much more of the science resources should be allocated to projects that have secured scientific diversity. Such projects usually have leaders with a demonstrated capacity to handle cognitive complexity by integrating different disciplines and fields.

Finally, it seems especially important from a regional development perspective that most areas outside north-western Europe increase their spending on basic science and applied research.

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