General Introduction

This book is placed within a long tradition of formal, mathematical analysis of Marxian economics, and indeed aims to revive it. Two related streams of literature are directly relevant to our project. The first stream concerns Marxian value theory, specifically the relationship between values and prices and the labor theory of value. For Marx values are the amount of labor time socially necessary to produce—embodied in—a commodity and serve as underlying regulators of the structure and dynamics of market prices. The labor theory of value purports that there is a direct correspondence of prices to values, but this idea has run aground on a series of mathematical and theoretical issues: the so-called “transformation problem”. The transformation problem has generated a vast literature with contributions from those trying to salvage Marx’s theory as it is, those trying to show its unescapable defects, and those attempting to provide coherent reinterpretations in the spirit of Marx’s original work.

The transformation problem and the associated issues are not new, with the debates around it beginning shortly after Marx’s death. Discussions of the transformation problem make cyclical appearances in the literature, with long periods of relative inactivity punctuated by spurts of heated debate. One of the pioneers of a rigorous approach to value theory, and a central figure in the first wave of debates, is Ladislaus von Bortkiewicz (1949). His analysis of the relation between prices of production and labor values can be considered as the starting point of the rigorous treatment of the transformation problem. This is not the place to provide a comprehensive discussion of von Bortkiewicz’s contribution and of the subsequent literature. For our purposes, the key point is that von Bortkiewicz showed that, even in the simplest linear economies, labor values—as traditionally defined—cannot be thought of as determining (relative, equilibrium) prices in any meaningful sense. Thus, the (standard interpretation of the) LTV is flawed.

It is important to stress that von Bortkiewicz’s results are mathematically true, and in a sense, von Bortkiewicz set the parameters...
of the discussion, becoming the point of reference—positive or negative—for most of the subsequent contributions. Some authors have denied the relevance of von Bortkiewicz’s analysis arguing that he focused on the wrong price or value magnitudes, and that appropriately defined values do determine appropriately defined prices. More persuasively, other authors have interpreted von Bortkiewicz’s results as the starting point for a reconsideration of the role of value theory within Marxian economics. The LTV may be inadequate to explain relative prices,¹ but this does not mean that labor accounts are irrelevant to explain some fundamental characteristics of capitalist economies that are relevant from both a positive and a normative viewpoint.

The key point to note is that there is no single, natural interpretation of Marxian value theory. The received view is that the main aim of the LTV is predictive: labor values are meant to explain (relative, equilibrium) prices. Yet even within a predictive interpretation, labor magnitudes may be relevant to explain other phenomena of capitalist economies. For example, one may argue that the LTV establishes a relation between profits and exploitative relations, thus allowing one to explain investment and growth. More generally, however, it is not clear that Marxian value theory can only be interpreted as a predictive exercise. For “there are at least three distinct non-metaphysical interpretations of the labour theory of value, viz. (i) descriptive, (ii) predictive and (iii) normative” (Sen 1978, 175).

One descriptive interpretation of the LTV is that of capturing the process of formation of equilibrium prices in capitalist economies, as in the standard view. But this is certainly not the only possibility. One may argue that in the LTV “it is the activity of production that is being described...[with a focus] on ‘personal participation’” (Sen 1978, 177). Or it may be relevant to measure and understand some important characteristics of capitalist economies—such as the dynamics of productivity and profitability, or the implications of technical progress—that are not immediately visible by focusing on monetary, market magnitudes.

¹ It is worth noting, however, that the second important, albeit often neglected, contribution of von Bortkiewicz’s seminal analysis is the early solution to the problem of the ‘transformation’ of labor magnitudes into price magnitudes based on the proof of the existence of a clear relation between the sphere of labor accounts and the sphere of monetary accounts. For a comprehensive discussion and review of the literature see Desai (1988). For more general, formal analyses see Morishima (1973, 1974) and Roemer (1981).
But the LTV can also be interpreted primarily as providing the foundations for a normative, evaluative exercise and an indictment of capitalist relations of production. For example, one may argue that it explains the origin of profits as accruing from the exploitation of workers and therefore shows the illegitimacy of capitalist income, and the source of significant inequalities of well-being. Or it may be taken as providing the foundations of a distributive approach based on contribution and effort.

None of these interpretations are metaphysical, and none of them are affected by the standard criticisms leveled against Marxian value theory based on the transformation problem.\(^2\)

The second stream of literature focuses on the Marxian theory of exploitation and classes. In the early stages of the Marxian economics literature, the solution of the ‘transformation problem’, and the proof that relative prices are determined by labor values was considered to be central to the Marxian project of proving that capitalism is fundamentally exploitative. One of the major contributions of Nobuo Okishio is the proof that—in linear, two-class economies with homogeneous labor—aggregate profits are positive if and only if the aggregate rate of exploitation is positive (Okishio 1963). In other words, even setting aside the issue of whether labor values determined relative prices, it is possible to prove that capitalism is inherently linked with the exploitation of workers.\(^3\) Indeed, the result has often—somewhat misleadingly—been interpreted as proving the Marxian insight that exploitation is the only, or main, source of profits. The significance of this result is such that it has been dubbed the Fundamental Marxian Theorem (FMT), and it has sparked a substantial literature.\(^4\)

The contributions by von Bortkiewicz and Okishio, and the subsequent literature, allowed the establishment of some key insights of Marxian value theory—albeit properly qualified—in simple, two-class linear economies. Yet they left open the fundamental question of

\(^2\) For a detailed analysis of the variety of interpretations of Marxian value theory and a novel axiomatic interpretation of the LTV see Mohun and Veneziani (2017).

\(^3\) The result has been successfully extended to Leontief economies with heterogeneous labor by Bowles and Gintis (1977), Krause (1981), Fujimori (1982), and Veneziani and Yoshihara (2011).

\(^4\) The literature is too vast for a comprehensive list of references. In addition to the classic contributions cited in the text, it is worth mentioning the more recent discussions by Fleurbaey (1996), Mohun (2003), Flaschel (2010), Veneziani and Yoshihara (2012), and Veneziani and Yoshihara (2015a). For a survey see Yoshihara (2017).
the validity of their results in more general settings. The key claim of Steedman’s (1977) famous analysis is precisely that, outside of the simplest Leontief economies, not only are labor values irrelevant to determine prices of production: labor accounts are irrelevant to understand capitalist economies, labor values are logically flawed and the notion of exploitation is at best useless. Steedman (1977) analyzed von Neumann economies characterized by joint production and showed, by means of examples, that in this more general setting labor values could turn out to be negative and that the FMT does not hold: it is possible to have positive profits with negative surplus value and thus a negative exploitation rate.

These conclusions sparked a new wave of debates. Given the rather heated controversy, and the sometimes unfair criticisms (motivated by the perceived political relevance of the conclusions that seemed to question the core of Marxian economics), it is worth stressing that Steedman’s results are plain mathematical truths. Within the formal and conceptual framework adopted by Steedman, there is no way of escaping his conclusions, and any criticisms denying this are a plain red herring. Rather, Steedman’s seminal contribution can be interpreted as clarifying the terms of the analysis. Certain simple-minded generalizations of concepts that hold in Leontief economies are inappropriate in more general settings and yield paradoxical conclusions. Consider the notion of labor values first. In simple Leontief economies, the labor values of commodities are the standard Leontief employment multipliers. Steedman’s key assumption is that this definition of labor values holds also in more general settings. This is by no means trivial and many authors have criticized this approach on formal, methodological and even exegetical grounds. Morishima (1974) and Morishima and Catephores (1978), for example, rejected Steedman’s definition because it allowed for technically inefficient processes to determine labor values.5 More radically, since the early 1980s, the ‘New Interpretation’ originally developed by Duménil (1980) and Foley (1982)6 has questioned the traditional dualist interpretation of Marxian value theory that has dominated the debate from the publication of Capital Volume III up until the 1970s—including both

5 Morishima (1974) and Morishima and Catephores (1978) define the labor embodied in a bundle of goods as the minimum amount of labor necessary to produce the bundle as net output among all existing production techniques.

Steedman (1977) and Morishima (1973). In the dualist approach, money plays no role and labor values and monetary magnitudes are assumed to form two conceptually separate systems. There is an underlying (intrinsic, invisible, essential) system of labor values and associated exploitation, and a phenomenal (extrinsic, visible, superficial) system of prices and profit rate. Marxian value theory is then interpreted as a predictive tool that bridges the gap between the two systems: relative labor values are meant to explain equilibrium relative prices. According to the New Interpretation, instead, money and labor accounts are expression of the same set of underlying mechanisms and form a single system within the circuit of capital. Thus, it is possible to define a variable, called the monetary expression of labor time (the ratio of net national income over total labor spent in production), which acts as a conversion rate between value and price magnitudes, allowing one to move from monetary accounts to labor accounts and vice versa.

Both Morishima’s approach and the New Interpretation have raised inevitable controversy, but it is fair to say that their definitions of labor values in general economies are arguably closer to the spirit—if not the letter—of Marx’s own notion of labor values than the employment multipliers used by Steedman (1977). Further, by severing the link between the notion of labor values and that of employment multipliers, they avoid the paradoxical results derived by Steedman: negative labor values do not appear in either approach. Similar, and theoretically related, doubts have been raised on Steedman’s definition of exploitation as the difference between the amount of labor expended by workers and the value of labor power, where the latter is defined as the amount of labor embodied in the workers’ consumption bundle, computed using employment multipliers. According to Morishima

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7 Nor do they emerge in other approaches proposed in the literature sparked by Steedman’s (1977) book. The discussion here does not aim to provide a comprehensive review of the debate. Rather, it focuses on those contributions that are conceptually and/or methodologically closer to the approach laid out in this book. Other interpretations of value theory that reject Steedman’s conceptual and/or formal framework include Fine and Harris (1979); the ‘Single-System Interpretation’ by Wolff, Roberts, and Callari (1982); the ‘Temporal Single-System Interpretation’ (see for example, Freeman and Carchedi (1996), Kliman and McGlone (1999), for a thorough critical discussion see Veneziani (2004, 2005) and Mohun and Veneziani (2007, 2009)); the ‘Macro-Monetary’ interpretation by Moseley (2000, 2016); and the approach by Shaikh (1998, 2016). For a comprehensive survey, see Mohun and Veneziani (2017).
(1974), for example, employment multipliers should not be used to compute the labor embodied in the workers' consumption bundle. If his definition of labor values is used, instead, then the FMT can be shown to hold in the balanced growth equilibria of von Neumann economies. According to the New Interpretation, instead, the value of labor power should be defined as the monetary wage divided by the monetary expression of labor time and not—as in standard dualistic approaches—as the labor embodied in the bundle consumed by workers. Using this notion of the value of labor power, it is not difficult to show that the perverse examples provided by Steedman (1977) do not arise and a robust relation between profits and the exploitation of the working class can be shown to hold in general economies (Veneziani and Yoshihara 2012, 2015a, 2017b).

In summary, Steedman’s contribution is important and it has significantly helped to clear the ground from erroneous interpretations. Yet it is not the last word on Marxian economics. The debate around and after Marx after Stauf has helped to rigorously pose, and address, some fundamental questions concerning the role, scope and fundamental tenets of Marxian value theory. This book aims to revive and contribute to this tradition.

To be specific, we explore and develop an interpretation of Marxian value theory originally proposed by Flaschel (1983a). Conceptually, this approach is analogous to other recent interpretations of Marxian economics, especially the New Interpretation, in its emphasis on the monetary dimension of modern capitalist economies and on the relevance of an empirically grounded notion of values. Marxian economics, and especially Marxian value theory, is not relegated to the sphere of pure theory. It is a practical tool to understand the deep dynamic forces underlying capitalist development.

The solution to the so-called ‘transformation problem’ offered in this book is, conceptually, very simple. Marx’s labor values are an input-output (IO) based accounting construct and should be defined within a

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8 Yet Roemer (1981) showed that if Morishima’s (1974) definition is adopted, the FMT does not hold, in general, in economies with a convex cone technology, if a different notion of equilibrium is adopted, namely that of a reproducible solution (Roemer 1980a). Later, Roemer (1982) proposed an alternative definition of exploitation, which according to him would generalize the FMT (and other Marxian propositions) to convex cone economies at reproducible solutions. Veneziani and Yoshihara (2012, 2015a) have shown that neither Morishima’s nor Roemer’s definition preserve the FMT in economies with a convex technology and homogeneous labor.
given accounting system. In particular, as in Richard Stone’s “System of National Accounts” (SNA), labor values are simply the total labor costs defined in the SNA. Thus, as soon as joint production is allowed for, there can be no purely technological definition of labor values. Labor values constructed in this way do not determine relative prices. They are measures of real costs of production focusing on human labor, in order to describe and understand the deep structural characteristics of production in capitalist economies beneath the surface of market phenomena. They are an integral part of a SNA, whose aim is not to explain prices, but to define real growth and inflation rates at the aggregate level, the dynamics of real labor productivity at the sectoral level, and the fundamental determinants of profitability.

From this perspective, the contributions of Sraffa (1960) and the neo-Ricardians (including Steedman), while important, are all but the last word on Marxian economics, and the so-called ‘transformation problem’ can be considered an empty expression. Rather than thinking in terms of Marx after Sraffa, at least as far as Marx’s LTV is concerned, we would say that Marx after Stone may be the proper slogan for future research.

Themes of the project

For the Classicals, observable magnitudes and events are more often than not mere epiphenomena of deeper structures, and a focus on market outcomes would yield at best a preliminary, superficial understanding of economic processes. This is not to deny their relevance in everyday economic life. The market prices of commodities (including labor), for example, are observable and important magnitudes in determining agents’ decisions, the survival of firms, government policies, and so on. However, according to the Classicals, market prices are influenced by a large number of unpredictable and short-lived influences and thus are not very informative of the deep underlying structures of capitalist economies. Thus, although we will discuss market prices, this book will introduce and analyze the classical notions of values and exploitation, and of ‘natural’ prices.

In the classical perspective, observable market prices are the effect of largely unpredictable and impermanent events, but their fluctuations usually occur around some stable magnitudes, whose determinants can be investigated and whose values can be predicted. Such centers of gravitation of the dynamic processes in the economy are the ‘natural’ prices of commodities and the proper object of economic analysis. Different authors have provided different definitions of natural prices.
In the Sraffa-von Neumann approach, advocated by neo-Ricardians, for example, natural (or production) prices require both the existence of perfect second-hand markets for fixed capital goods (conceived of as joint products of production activities) and the equalization of profit rates across sectors on circulating capital. This approach is theoretically rigorous and formally elegant but several of its assumptions have been questioned. First, the treatment of fixed capital as a joint product does not properly reflect the actual behavior of firms. As argued by Bródy (1970) in his stock-flow analysis, fixed capital is arguably not a jointly marketable product. Further, and related, the notion of the sectoral rates of profit implied by the Sraffian approach neglects replacement investment and, perhaps more importantly, it relates profits to the flow of material inputs rather than to the sectors' capital stock that ties up the money invested.

In this book, we adopt a different concept of natural, or production, prices that builds on the seminal IO approach developed by Andras Bródy and Wassily Leontief. We shall focus, in particular, on Leontief's (1953) almost forgotten notion of a capital stock matrix, which emphasizes the notions of capital advanced, capital consumed, and the related turnover times as well as the importance of stock-flow accounting in general. The relevant profit rate, we shall argue, should be computed on the capital tied up into production processes. Further, we shall consider a deviation from the standard assumption of equalized profit and wage rates across production sectors. This assumption is motivated by the idea that competitive forces tend to predominate, at least in the long-run. Yet, the structure of modern capitalist economies is far from the competitive benchmark, and the empirical evidence on the existence of a tendency towards equalization is rather mixed. In fact, a hierarchy of profit and wage rates tends to emerge, and persist over time, across sectors. The question then arises whether the key insights of the classical-Marxian theory of production prices—such as the existence of a fundamental distributive conflict between workers and capitalists—continue to hold when the assumption of a uniform profit rate is relaxed. Therefore we explore a generalization of the standard model which incorporates fixed capital and a constant hierarchy of profitability across sectors.

The other key concept that will be introduced is that of (labor) values. This notion was prominent in classical economics, as it was thought to identify some deep structural aspects of the production process, independent of the vagaries of market forces. The subjectivist approach to value that has come to predominate in mainstream economics has relegated the analysis of value to the sideline of academic
General Introduction

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The received view (both in mainstream and, alas, in much of heterodox economics), in fact, is that the notion of labor value is at best redundant (Samuelson’s famous blackboard theorem) and likely logically incoherent, especially outside of extremely simple economies.

In this book, we propose a notion of labor values as the total labor costs of produced commodities, that is different—in general—from employment multipliers; is logically coherent and general; avoids paradoxical results; and is based on the actual accounting practices of capitalist firms. Then we show that the notion is relevant to understand some deep—if invisible—dynamics of modern economies, including the trends in (aggregate and sectoral) productivity and aggregate profitability. Far from being a metaphysical construct, labor values and labor accounts in general can be used in empirical studies of actual economies.

Based on this notion of labor values, we shall advance a general definition of exploitation that is also logically coherent (in general production economies) and relevant to understand the deep trends of actual capitalist economies. At the aggregate level, the rate of exploitation will be defined as the difference between the amount of labor performed by workers and the value of their labor power. Again, not only is this concept of exploitation based on well defined, empirically measurable magnitudes; it can also be shown to be one of the key determinants of the general rate of profit—arguably the most relevant variable in capitalist economies, as it is the main determinant of capitalist production and investment decisions.

In this book, we will not analyze the exploitation status of individual agents, nor will we discuss the normative relevance of the concept of exploitation. These are important questions which must be addressed in order to provide a general theory of exploitation. Nonetheless, our analysis should show that these questions are worth asking and, contrary to the received wisdom, a general, logically coherent and empirically relevant definition of exploitation can be provided which helps to understand modern capitalist economies.

Further, we focus only on the distributive aspects of the notion of exploitation in a static context. We examine neither the relevance of power and coercion in exploitative relations, nor the dynamics of exploitation. For a discussion of power and the persistence of exploitation, see Veneziani (2007, 2009a,c, 2013), Veneziani and Yoshihara (2015b), and Cogliano, Veneziani, and Yoshihara (2016). For a critique of merely distributive approaches, see Veneziani (2008, 2009b, 2012).

According to some critics, the notion of exploitation does not add much to our understanding of capitalist economies because it is just a complicated
Structures: One-, two-, and $n$-sectoral linear production models

The nature of our investigation is primarily theoretical: we aim to rigorously define some important concepts that allow one to analyze modern capitalist economies, and investigate their properties. For this purpose, we will set up and analyze abstract models of production economies. To be specific, we will focus on linear production models (IO matrices with constant returns to scale in each sector).

There are two main reasons for this choice. First, the vast majority of the literature in the classical tradition has indeed focused on linear models. Although our concepts and results hold more generally, it is useful to focus on linear economies in order to compare our work with the rest of the literature. Second, and perhaps more important, linear production models are at the core of the IO literature and, as we will argue in the rest of the book, there are many deep theoretical connections between IO theory and the classical-Marxian approach. In this context, linear models do not necessarily reflect assumptions on technology and substitutability between factors of production. Rather, IO matrices provide a theoretically informed ex post portrayal of the production side of the economy at the sectoral level.

For expositional reasons, we shall sometimes consider economies with only one, or two, produced commodities. This will allow us to introduce the reader to the most complex concepts of the book in the clearest possible way, by abstracting from all technical complexities and also to clarify the theoretical issues that arise by moving from one-good models to more general economies with multiple commodities. The transformation problem, for example, can be conceptualized precisely as an aggregation problem that arises when one considers more than one produced commodity.

It should be stressed, however, that our analysis of IO structures is not purely theoretical. In fact, the book contains numerous empirical illustrations of the main concepts and results, focusing in particular on the German economy at the beginning of the 21st century. More deeply, as we shall argue repeatedly in the course of the book, unlike in much of the mainstream and heterodox literature, we adopt what way of capturing the productivity of the economy. This objection is misguided and in the modern approaches to value theory, the existence of profits is not synonymous with the existence of a surplus denominated in any arbitrary commodity (as the Generalized Commodity Exploitation Theorem implies, see for example Roemer (1981)). For a discussion see Veneziani and Yoshihara (2010, 2013a).
might be called an empirically-informed theoretical approach, whereby empirical reality and stylized facts—e.g. concerning firm’s actual profit-maximizing practices—act as constraints on theoretical constructs, and guide research efforts. Thus, for example, our definition of labor values is based on the actual accounting and production practices of capitalist firms. We conceive of fixed capital as capital tied up in production, rather than presuming the existence of perfect second-hand markets for used machines. We explore the implications of persistent barriers to competition for production prices by explicitly considering intersectoral wage and profit rate differentials. And so on. From this perspective, the classical-Marxian approach developed in this book has natural applications to the analysis of empirical questions.

In Part I of the book, we analyze simple models with no joint production, uniform turnover time, and circulating capital only. This partly reflects the chronological order of our exposition: the simpler linear models are more appropriate to capture the theoretical framework of early classical authors. In Part II, we will relax some of these assumptions and deal with the complex issues raised by joint production, fixed capital (or, more precisely, capital tied up in production) and different turnover times in production.

Most of our analysis is conducted at the microeconomic level, and focuses on the intersectoral relations in general n-commodity economies, as we aim to show that a logically coherent and empirically relevant interpretation of Marxian value theory can be provided which meets the standards of analytical clarity and rigor of modern economic theory. A number of results are proved for the disaggregated n-good economy that provide interesting insights on the structure and laws of capitalist economies. We see our analysis as a first step in the development of a Marxian value-theoretic understanding of modern capitalism, and in the construction of a bridge between the microeconomic foundations of Marxian theory and the analysis of the macro-dynamics of capitalist economies. Indeed, this book can be seen as part of the construction of a broader theory inspired by the work of Marx, Keynes and Schumpeter.

Readership

The book aims to reach a wide readership. The topics analyzed should be of immediate relevance for scholars working in the classical and Marxian traditions, which this book explicitly aims to revive. However, some of the fundamental concepts (such as, among the others, the classical notion of competition and prices, the treatment of fixed
capital, the classical concept of equilibrium) and of the issues raised (for example, the efficiency of capitalist economies) should be of interest to all economic theorists. Indeed, our analysis is conducted using the standards of rigor of contemporary economic analysis and we adopt the formal, methodological approach common to much economic theory. General equilibrium theorists, mathematical economists and IO theorists, for example, should find many of the topics discussed in the book familiar.

It should be stressed, however, that our approach is not purely theoretical, and readers with more empirical inclinations should find some food for thought too. In fact, as noted earlier, the book contains numerous empirical illustrations of the main concepts and results, and the classical-Marxian approach developed in this book has natural applications to the analysis of empirical questions.

Although our aim is not exegetical, the themes developed in the book should appeal also to readers interested in the history of economic thought (and, more broadly, the history of economic ideas). To be sure, we shall not provide a detailed analysis of classical texts, nor do we aim to provide the correct interpretation of classical and Marxian economics. The main purpose of this book is to develop a conceptual framework inspired by classical authors, and especially Marx, that allows us to derive important insights on modern capitalist economies. We will not try to show that classical authors, or indeed Marx, “are right”: theories developed almost two centuries ago at the dawn of the capitalist era can hardly be expected to be valid today. Yet we hope that the analysis of classical concepts and theories with the modern tools of mathematical economics will not only provide a different perspective on these authors, perhaps shedding new light on their works, but it will also convince readers that there is much still to be learned in the classical and, especially, Marxian approaches.

In particular, we hope that students (PhD candidates, MSc and advanced undergraduate students) will find the book stimulating and useful. For this purpose, we have tried to make the book as accessible as possible. In particular, no previous knowledge of the theories and approaches developed in the book is required. The structure of the book allows every reader to follow the argument by building up in complexity and sophistication from chapter to chapter. Further, we have tried to keep the formal analysis at the simplest possible level compatible with the fundamental standard of clarity and generality of modern economic theory. For most of the models and results presented in the book, a basic knowledge of linear algebra is all that is required to understand the logic and implications of our arguments. In most cases, especially in
Part I of the book, we introduce relevant definitions and concepts and derive basic results first in the simpler one- or two-commodity settings and then show that they can be generalized in more general economies by means of matrix algebra.

But even when the concepts, models and propositions may seem forbidding, we hope that the readers, and especially the younger ones, will not be discouraged. In an era where mainstream economics, with its toolbox of representative agent models, efficient markets hypothesis, perfect competition assumptions, and so on, is in disrepute, our hope is that a new look at the Classicals and Marx can bring a breath of fresh air, and encouragement to search outside the box.

Plan of the book

Part I introduces the main classical authors. These chapters describe the historical roots and provide the context of our analysis. Sadly, modern mainstream economics behaves as a discipline without a history, often giving birth to old ideas. Contrary to this widespread attitude, Part I of the book explicitly shows the evolution of the fundamental ideas of our approach in history and places our analysis clearly within the history of economic thought. The chronological order of the chapters, however, also allows us to introduce the reader to increasingly complex and sophisticated concepts and tools of analysis.

Chapter 1 focuses on François Quesnay and his Tableau Économique, which represents the first attempt at constructing a complete formal model of the circular flow of income and of the relations between different sectors in the economy. In this sense, the Tableau Économique can be seen as the forerunner of modern IO analysis, and we shall indeed analyze it using the tools and concepts of IO theory. We characterize production structures which are productive and profitable, and define the concepts of productiveness and profitability for general n-sectoral IO tables interpreted as linear production technologies. We show the equivalence of these two concepts and provide some further characterizations focusing on Leontief-inverses and their use in multiplier analysis, which together demonstrate the dual structure (concerning quantities and prices) of IO analysis.

Having introduced the building blocks of the IO representation of economic systems in Chapter 1, Chapters 2 and 3 analyze Adam Smith (2000) and his notions of competition and prices. We focus on two central themes in Smith’s work, namely the increasing division of labor and its implications for commodity exchange and prices, and the (unintended) economic consequences of individual (economic) actions.
We start from Smith’s “early and rude state of society,” a pure labor and corn economy. In this context, the relationships between the total labor costs embodied in the various commodities, the natural prices that guarantee a uniform rate of profit on the basis of a uniform wage rate, and labor commanded prices, representing purchasing power in terms of labor, can be clearly seen. We then use again Quesnay’s simple structure of production in order to see how normal prices, labor costs and labor commanded prices can diverge once produced means of production enter the picture. Then we prove that the strict pursuit of self-interest does not lead to economic and social chaos, but instead—as Smith claimed—to the maximum corn production that is possible with given labor resources and given land of decreasing fertility (as assumed by the classical authors). In other words, the classical notion of ruthless competition provides a theory of long-period prices as the center of gravity for market prices, and it allows us to show that, under certain conditions, the ruthless pursuit of self-interest by individual agents may lead to maximum production.

In Chapter 4, we turn to David Ricardo (1951) and his theory of natural or long-period prices of production (and the underlying wage-profit curve), balanced growth paths (and the underlying consumption-growth curve), and technical change in a manufacturing environment. We will investigate questions of changing income distribution, their implications for price formation and the choice of technique and balanced economic reproduction, from a long-period perspective, in order to understand some fundamental relationships that characterize the process of capital accumulation and price formation in a capitalist economy.

We provide tools for an economic analysis of price-quantity relationships that question (i) standard results of the neoclassical theory of economic growth and distribution, (ii) a narrow and one-sided understanding of labor values as the main force that drives price formation, and (iii) the Sraffian interpretation of the Standard Commodity as a means to exploring the causes of changing relative prices of production due to changes in income distribution or even technical change. The attempts to construct ‘real’ magnitudes (be it capital, embodied labor, or invariable measures of value) behind the surface of nominal price-quantity expressions therefore demand closer examination.

Chapter 5 extends the analysis of Chapter 4 to explore the von Neumann model with joint production and multiple activities, and to examine the dynamics of prices and quantities. We analyze a model of the gravitation of market prices around classical prices of production.
and balanced growth. The dynamics need not be convergent and it can be formulated in a very general way, in a rectangular system with joint production that allows for process as well as product extinction in general $n$-good economies. We show that Sraffa/von Neumann production prices are centers of gravity for market prices, but convergence may take a rather long time. Chapter 5 concludes the analysis of the classical authors.

Having introduced the basic definitions and concepts, and the main theoretical tenets of classical economics in Part I of the book, Part II engages with Karl Marx’s contribution and, in particular, with his LTV. Each chapter deals with different aspects of Marx’s theory considering the challenges that multiple activities, joint products, fixed capital, and so on, pose to the LTV, while at the same time showing some limitations of the standard classical approach to prices and values. The starting point of Part II is two well-known results in mathematical Marxian economics. First, as shown in Part I of the book, apart from very special cases, production prices are not proportional to labor values. Therefore, contrary to the standard reading of the LTV, equilibrium prices are not determined by labor values. Second, and perhaps more worryingly, Steedman (1977) famously showed that, outside of the simplest Leontief economies, labor values—defined in the conventional fashion as the standard employment multipliers—and surplus value can be negative. In the light of these results, two related questions immediately arise: if they are not good predictors of equilibrium prices, what are labor values good for? And, in any case, does the concept of labor values only make sense in the simplest linear economies?

Rather than presenting our definition of labor values straight away, in Chapter 6, we start from first principles and discuss the role of labor values (‘what labor values are good for’) and, consequently, the properties that a proper, general definition of labor values should have. We argue, among other things, that the appropriate definition should be such that labor values are always nonnegative, and the labor value of any commodity is zero if and only if its price is zero. Further, different types of labor should be homogenized using wage rates as the relevant conversion rates. We also state some results that hold in simple Leontief economies, including a relation between the existence of exploitation and positive profits at the aggregate level (the FMT), the proportionality between prices and values when the rate of profit is zero, and the aggregate equality between total direct labor spent in production activities and the labor value of the net product. We argue that these relations represent the core of Marxian value theory and any
appropriate definition of labor values should preserve them.\textsuperscript{11} It may be objected that these relations should be, and indeed usually are, proved as results in a given economic environment, under certain conditions. Yet the central relevance of each such relation in value theory is such that “its epistemological status in our understanding is as a postulate” (Roemer 1982, 152).

In Chapter 7, we examine the role of labor values and labor accounts in economic theorizing. We provide a comprehensive analysis of technical change and the notion of labor productivity both at the sectoral and at the aggregate level. We analyze the conventional indices of labor productivity used in systems of national accounts (SNA) based on notions of sectoral real value-added per unit of labor and the IO productivity measure, namely the reciprocal of the IO employment multipliers that is, the labor values of Marxian economic theory. We show that the latter provide the theoretically sound measures of sectoral and economy-wide labor productivity, with purely technological foundations insofar as IO coefficients can be interpreted as pure quantity magnitudes.

First, a unified theoretical framework for the analysis of productivity measures is provided, which is based on a novel axiomatic method. Rather than comparing different measures in terms of their implications in various scenarios, we start from first principles and formalize some theoretically desirable properties that any measure of labor productivity should satisfy. The main axiom focuses on changes in productivity and states that labor productivity at time $t$ in the production of good $i$ has increased relative to the base period, if a unit increase in the net product of good $i$ demands less labor than in the base period. This is a weak restriction which incorporates the key intuitions behind the main productivity measures in the literature. Yet it characterizes the IO measures, whereas the conventional SNA indices do not satisfy it in general owing to their inherent dependence on relative prices and final demand. The second major contribution of this chapter is a rigorous analysis of the conditions under which profitable innovations lower labor values, thereby raising productivity and increasing consumption and investment opportunities. In a model with fixed capital and possibly differentiated sectoral profit and wage rates, we derive a theoretical foundation for the \textit{Law of Decreasing Labor Content (LDLC)}. The inherent functioning of the capitalist

\textsuperscript{11} For a discussion of the role of formal models in Marxian economics, see Veneziani and Yoshihara (2017b) and in economic theory in general, see Mohun and Veneziani (2012).
system, in particular the forces driving technical change, including class struggle, leads to a tendential decrease in the amount of labor necessary to produce (or, indeed, embodied in) commodities. To be specific, profitable fixed-capital-using labor-saving innovations lead to productivity increases. Given that capital-using labor-saving technical change has characterized most of the phases in the evolution of capitalism, this result provides theoretical foundations for the conclusion that labor values tend to fall, and labor productivity tends to rise, over time in capitalist economies.

In Chapter 8, we extend our analysis and show that labor values—or total labor costs, in our interpretation—provide important insights on one of the most important issues in Marxian economics, namely the determinants of the general, or economy-wide, rate of profit, and the LTV explains the key variables determining profitability. More generally, far from being metaphysical constructs, labor values can be used to investigate empirically some of the fundamental dynamic laws and tendencies of capitalist economies. Thus, we introduce the notions of capital consumed and capital advanced in production and discuss Leontief’s notion of a capital stock matrix, and how to compute it based on the available IO data. This is important because profitability should be measured in relation to the stock of capital advanced, and tied up in production both at the sectoral and at the aggregate level.

We identify three main determinants of the value rate of profit: the creation of absolute and relative surplus value, and technical change and accumulation (via their effects on the value of the total capital stock). We show that differences between the value rate of profit and the price rate of profit depend on the differences between the wage share and the value of labor power, and between the price and value measures of the total capital stock. Then, empirically, we show that the LDLC holds and it affects the evolution of aggregate profitability in the German economy 1991-2000. We also show that, although prices and labor values of individual commodities may deviate, at the aggregate level any such differences are irrelevant and the key insights of the Marxian theory of exploitation and of the profitability of a capitalist economy are valid.

In Chapters 7 and 8, we assume that each sector produces a single commodity by using one method of production, and so labor values can be defined as the standard IO employment multipliers and have all of the usual, desirable properties. Chapter 9 extends our analysis to production economies in which each sector produces a single good but multiple activities are used to produce the various goods. The key step for the generalization of the concept of labor
values as measuring the real (labor) costs of producing goods is to note that the existence of alternative methods of production requires, following Marx, distinguishing (average) labor values from individual values. The distinction between individual and average requirements is standard in IO methodology, which derives a conventional square IO table by aggregating the activities of the given sectors using the activity levels that characterize the single activities into some suitably defined ‘average’ technology $A',l'$, which is derived from the original IO structure $A,l$. Labor values are then defined as the solution of $vA' + l' = v$, and represent the average total labor costs of producing the various commodities with respect to the multiple activities that are operated in each sector. We show that this extension of the concept of labor values has all the desired properties and it allows us to generalize the standard propositions of Marx’s economics.

The presence of multiple activities requires the introduction of the distinction between individual and (average) labor values, but Marxian value theory can still be analyzed based on merely physical, production-based quantities. If joint production is considered, a purely technological definition of labor values is not necessarily appropriate theoretically and it is quite distant from the actual accounting practices of profit maximizing firms. In Chapter 10, alternative methods of determining the total labor costs, or requirements, of commodities both in money value and in physical terms are discussed and different extensions of the definition of labor values are considered based on the actual accounting practices of capitalist firms. We argue that it is conceptually impossible to separate price and quantity magnitudes. We argue that, the appropriate way to disentangle the joint outputs of each sector or activity is by using the so-called “industry technology hypothesis” which splits up all inputs in proportion to the relative value of output in the output basket of the joint production activity. In the full-cost accounting techniques of business administration, this is called the “sales value method”, since the relative proceeds of the items in a joint bundle determine the amount of joint inputs these single items have to bear.

In Chapter 11, we show that once the impossible quest for pure physical values is abandoned, it is possible to provide definitions of labor values—actual labor values—that are theoretically robust and empirically meaningful, and that preserve the main propositions of Marxian value theory. To be specific, we show that if jointly produced outputs and their inputs are disentangled by means of economic imputations which reflect the benefit received from each unit of costing—i.e. using the “sales value method”—then individual
and average labor values can be defined as in the case with multiple activities based on the square input matrix (and its corresponding labor input vector) obtained from IO methodology in the case of the industry technology hypothesis.

We show that actual labor values are well defined, positive and unique, and preserve the main properties of Marxian value theory in linear economies with joint production, including the key propositions on price-value relationships, such as the FMT. In fact, actual labor values display none of the paradoxical features shown in Steedman’s (1977) famous examples. We argue that such counterintuitive results derive entirely from Steedman’s definition of labor values as the standard employment multipliers—a definition which is conceptually inappropriate in the case of joint production or multiple activities.

The last two chapters deal with two further extensions of price and value theory. In the classical long-period approach, production prices entail the equalization of profit rates and wage rates across sectors. Conceptually, this is meant to reflect the idea that prices of production are centers of gravity of market prices and emerge due to competitive pressures that tend to remove any sectoral differences in wage and profit rates. The starting point of Chapter 12 is the acknowledgement that, empirically, there is remarkably little evidence of a strong tendency towards profit rate or wage rate equalization. Actually, a number of empirical studies, and our own analysis of the German economy 1991-2000 suggest that intersectoral differences are persistent and indeed show a remarkable stability over time. Therefore we extend the classical production price equations to incorporate the existence of persistent, and stable, wage and profit rate differentials. We show that, under the usual assumptions on technology, the generalized production prices are unique, well-defined and strictly positive. Moreover, all of the standard results of production price theory—including the existence of a negative relation between wages and profits—continue to hold.

In Chapter 12, we drop the standard assumption of uniform wage and profit rates, but the model is otherwise standard. In particular, fixed capital is ignored and the profit rate is computed as the rate of return only on so-called circulating capital. Chapter 13 extends the analysis by considering the implications of imperfect competition for the classical analysis of technical change, choice of technique, and distribution, from both a theoretical and an empirical perspective in economies with fixed capital. We reconsider the classical notion of prices of production when both of the standard Sraffian assumptions are dropped, namely profit rate and wage rate equalization and the presence of perfect second hand markets for capital used in production.
(and treated as a joint product): both assumptions are at odds with the actual features of modern capitalist economies. In contrast, we define classical prices of production assuming the existence of persistent wage and profit rate differentials across different sectors and incorporate the notion of fixed capital as capital tied up in production, using Leontief’s notion of capital matrices. We derive the so-called wage-profit curves in this more general framework and prove that the main classical-Marxian insights concerning technical innovations and distribution remain valid in this more general setting. Then, we analyze data from the German economy at the beginning of the 21st century to show that our general theoretical framework is more useful for empirical analyses than the standard neo-Ricardian approach. Interestingly, our analysis shows that the empirical wage-profit curves of the German economy (2000-2010) are fairly close to straight lines, independently of the choice of scaling, which confirms a host of similar results in the literature.

The analysis of imperfect competition and fixed capital takes us at the cutting-edge of the modern approach to Marxian price and value theory. We point out some directions for further research in the spirit of this book in the concluding chapter.