## Index

Agenda 21 113  
‘aggregative approach’ 41  
almost cooperative solution 106–10  
arbitrage argument 39  
asset accumulation equation 140  

Bachelier, Louis 149  
bang-bang control solution 13  

basic economy with transboundary environmental problems  
model 115–16  
numerical results 116–21  
sensitivity analysis 121–2  
benchmark  
independent price index 47  
invariant price index 45–6  
Brock model 9, 37  
money metrics/utility metrics 30–32  
non-autonomous optical control problem 10–13  
social optimum and externalities in decentralized economies 16–30  
stochastic version of 160–64  
Brownian motion process 148–53, 161, 168  
cost–benefit rule 164  

capital  
accumulation 17, 29  
concept of 4  
depreciation 115  
income tax 71, 78, 79, 81, 85–7, 89  
input 13–14  
investments 19  
shadow price of 20, 23–5, 146  
capital market, restrictions on 88  
capital stock  
current value shadow price 21  
equation of motion for 140–41  
growth under uncertainty 153  
probability distribution of 168  
as state variables 10, 80  
and technological progress 15  
carbon dioxide pollution  
data and parameters 125–6  
decomposition of welfare measures 126–8  
model 123–5  
results 128–31  
co-state  
equation 35, 163–4  
variable 18, 24, 82  
Cobb–Douglas  

instantaneous utility function 59, 124  
production function 59, 115, 168  
command optimum problem/solution 22, 30  
competitive path 21–3  
comprehensive net investment 44–5  
conditional equilibrium 99, 100–104, 110–12  
consumer  
assets 75  
behavior 123  
demand 107–8  
maximization problem 19–20  
preferences 31  
surplus 36–7, 39, 74  
utility function 80  
consumption  
comprehensive concept of 1  
external effects in 122–3  
marginal utility of 24–5, 104  
possibilities 2  
present value of future changes 147  
single homogenous good 16  
utility value of 27, 77  
consumption–investment pair 37–8  
continuous-time  
stochastic processes 148–52  
stochastic Ramsey model 152–60  
control  
functions 11, 155, 156  
problems 109  
process 161–2  
variables 10, 82
controlled market economy, welfare in 53
convex
differentiable cost function 17
feasibility set 13
cooporative solution 96–9, 100–101, 105–6, 110–12
cost–benefit analysis 9, 12–13, 25–30
cost–benefit rule 9, 17, 42–3, 51–3, 85–9, 102–6, 164–5
current value function 160
decentralized economies
applications 27–30
cost–benefit analysis 25–7
distortionary taxes in 75–7
externalities in 19–25
social accounting in 58
social optimum in 16–19
decentralized equilibrium 82
decentralized market solution 21–5, 100, 107
decision variables 10
differentiable value function 86–8
differential
equations 14, 26, 35, 39, 93, 169
games 8, 94, 134–9
discontinuities 12–13, 57, 109
discounting 10–11, 45
discrete estimation procedure 109
distorted market economies 2
distortionary taxation 7, 89
decentralized economies 75–7
green NNP analogue in second best
environment 79–85
model and first best welfare measure 71–5, 77–9
distributional objectives 144–7
Divisa price index 44
domestic
pollution 95–6, 107, 121
welfare 101
domestic producers, release of emissions 105
dynamic
budget constraint 20
equilibrium models 6, 13, 81
tax 23–4
dynamic global economy models,
numerical applications 113–14
basic economies 114–22
pollution and external effects in
production 122–32
economic
behavior 2, 32
sustainability 44–5
Edgeworth, Francis Ysidro 41
emission
function 29
permits 22
production function 72
technology 37
emission taxes 78–9
changes in 87–8, 105–6
design and implementation 48–68, 108
effect on employment 89
welfare consequences of 101–4
emissions
domestic producers 105
incentive to release 117
marginal cost of 88, 117, 130
rich countries 125–6
shadow prices of 16
socially optimal levels 16, 19
suboptimal generation of 102
employment 89, 139, 141–3
endogenous growth theory 13, 15
energy
cost function 115
first order conditions for 73
input of 16–17, 20
producer 72, 75, 76
envelope theorem 43
environment
assimilative capacity of 16, 27, 164
marginal damage to 104
environmental damage
effects of 123
from production 111
environmental policies
coordination of 92
effects on production decisions 104
environmental quality 106, 132
function 65
improvement in 98
indicator 93, 115
marginal utility of 117
relationship with pollution stocks 122
environmental taxes
benefits of 22, 98
Hamiltonian
  augmented 19
  current value 3–5, 21–3, 98–9, 141–3, 146–7
  country-specific 121
  of first best problem 29, 53–8
  generalized 158, 160, 167–8
  maximized 43
  present value 18–19
  pseudo 29, 32, 58, 65–7, 77–9, 108–9
human capital 59–60, 131, 132
imperfect competition in labor markets 139–43
income, marginal utility of 32, 37, 39, 40–41, 43
index numbers 41
indirect utility function 81
instantaneous social welfare function 144–5
instantaneous utility function 61, 65, 71, 93, 124, 139–40, 144
linear transformations of 31, 32, 74
money-metricized 36–42
interest
  on present value of future utility 115
  rate 75, 76, 81
intertemporal
  budget constraints 23
  economies 13
  index comparisons 37
investment, single homogeneous good 16
iso-elastic function 143
Ito
  calculus 148, 151, 168–9
  integrals 164–5, 168
Ito’s lemma 153–4, 158, 163
Jensen’s inequality 151
labor
  endowment 16
  force, growth of 153–4
  income taxation 71, 78–9, 81, 85–7, 89
  input 13–14
labor market
  competition 139–43
  restrictions on 88
Lagrange multiplier 81–2
leisure, social marginal value of 74, 139, 141–3
linear approximation 25, 74
linear emission production function 116
linear money-metricized index 35–6
linear transformations, instantaneous utility function 31–2
Lipschitz-continuity condition 135
lump-sum taxation 81
transfers 20, 23
marginal cost
  of energy emissions/use 117
  of utility value of consumption 27
marginal environmental damage, monetary cost of 25
marginal externality, present value of 21–3
marginal product of labor, utility value of 139
marginal productivity 20
marginal technological progress 32, 160
marginal willingness-to-pay 22, 98
market
  baskets 40
  data 22, 24
  imperfections 2, 4–5, 6, 9, 131
market economy
  Pigouvian view 100–101
  special cases 104–6
  tax reform in the conditional equilibrium 101–4
market solution 21, 24–5, 28–9
Markov
  control 165
  property 150
maximization problem 11, 19–20, 22
‘missing information problem’ 101, 108
money discount factor 39
money-metricized instantaneous utility function 36–42
money-metrics
  consumer surplus in 39
  cost–benefit rules 42–3
  genuine saving 44–6
  green NNP in 74
  measure of welfare 24
real NNP growth and welfare 44
relationship between GCNNP growth and genuine saving 46–7
static welfare measurement in 30–32
monopoly union wage setting 142–3
multi-country economy 111
multi-region numerical general equilibrium model 125
multidimensional production possibility set 37

Nash feedback-loop games
differential games 136–9
model 134–5
non-cooperative differential games
91–2, 114–16
welfare measurement under feedback control 135–6
national accounts, augmentation of 59–61
national welfare 122
measures 96, 98–9, 120, 121
negative externalities 15
neoclassical
growth model 152
production function 37
net interest rate 81
net investments
measure of 1
physical capital 125
vector of 43
net national product (NNP) 1–2
No Ponzi Game (NPG) 20, 49, 50, 75–7, 142
non-arbitrage condition 18–19, 23, 26, 28, 30, 38–9
non-autonomous
dynamic growth model 9
optimal control problem 10–13
technological progress 13, 14, 15
time dependence 9, 13, 83, 98, 129, 139, 154–6, 168
non-cooperative
Nash equilibrium 116–17, 121–3, 125–8, 130
Nash open-loop game 105, 106
Nash open-loop solution 94–6, 99, 100–101, 110–11
non-Pigouvian taxes 48, 50–53, 67
numerical general equilibrium model
59–60, 87
objective function 10–11, 14, 75
of representative consumer 28–9
one-country economy 111
open-loop strategy 114–15, 116, 135, 136, 137
optimal consumption policy 154, 156, 158
optimal consumption-saving decision under uncertainty 152
optimal control
problem 9, 137, 138, 165
theory 11
optimal path, conditions for 11, 18–19, 31
optimal taxation 76
optimal value function 25–30, 31–2, 74, 86, 102, 156–7, 159
optimization problem 11–12, 17, 21, 23, 79–83, 97, 146–7
ordinal utility function 31
ordinary least squares 125
output per capita, rich countries 125–6
parameter
values 116, 126
vector 25
parametric change 9
partial differential operator 163
perfect competition 4, 20
perfect foresight market economy 15
perfect market economy 9
permit markets 22
physical capital 75
marginal utility value of 56
net investments in 72, 115, 125
shadow price of 63–4, 143
stock of 93, 100, 126
Pigouvian taxes 22–4, 28, 92, 98–101, 109, 112
approximation of 6–7, 53–8, 61–6, 100–101, 106–10
policy
instruments 82–3
mix 70, 89
projects 2–3
rules 70
policy reform 85, 117
welfare effects of 70–71
pollute, decision to 100
Index

stochastic
  Brock model 160–64
  control theory 168–9
  processes 148–52
  Ramsey model 152–6
  welfare measure 168
stock externalities 101, 107
stock option pricing theory 149
stocks, net changes in 1–2
Stratonovich integral 168
suboptimal
  distribution 144, 147
  equilibrium 85
  value function 28–9
Swedish Energy and Environment Policy (SEEP) model 61, 62–3
tax
  bases 86–7
  mix 71, 85, 89
  policy 110
  rates 80, 82
tax reform 85, 92, 102, 106, 111
  in the conditional equilibrium 99, 101–4
tax revenues, lump-sum transfer 23–5, 74–5, 100, 102
technological change 128–9, 131, 132
  influence on production 124
  rate of 126
  value of 60–61
  welfare contribution of 2, 129, 130, 131
technological progress 15, 32
  non-attributable 159–60, 168
  rate of 14
  relative importance of 32
  shadow value of 18
  uncertainty about 152
  welfare contribution of 32
time dependent
  discount factor 10, 11, 13
  lump-sum transfer 20
time preference 19
transboundary environmental problems 7, 114–15
  model 115–16
  numerical results 116–21
  sensitivity analysis 121–2
  transversality condition 20, 24, 27, 84, 100, 162, 164
  two-country
    global economy 92–3
  two-point boundary problem 137–9
uncertainty, welfare measurement under 8, 156–60, 167–9
  continuous-time stochastic processes 148–52
  continuous-time stochastic Ramsey model 152–6
  cost–benefit rules 164–5
  HJB equation 165–7
  stochastic Brock model 160–64
  uncontrolled market economies 16, 19, 110, 114–16, 117, 120–21, 123, 130–31
  equilibrium in 116–21, 125, 127–9, 130
  welfare in 53, 122
  unemployment 139–43
  uninternalized external effects 59, 79, 96, 111, 122, 130
unit tax 22
United Nations Conference on Environment and Development (UNCED 1992) 113
utilitarian social welfare function 124
utility 35–6, 39
  distribution across individuals 144
  maximization, first order conditions 31
  metrics 30–32
  present value shadow prices of 94
  revenues in terms of 27
  value of 18–19, 57
utility discount factor 39, 78, 83, 98, 145
  rate 83, 88–9, 120–22, 126, 129, 135, 139
utility functions 39–41, 67, 74, 80, 86–7, 111, 144, 155–6
  first order approximation of 24
  linear approximation of 34–5
  maximization of 13, 102
  monotone transformations of 31, 36
  non-linear 110
  parameters of 126
  properties of 123
utility metrics, welfare measurement in 30–32
value function 86–8, 109, 165
   envelope properties of 94
wage rate 74–6, 139–43
wage setting 142–3
wealth 20
   marginal utility of 79–81, 83, 88
   shadow prices of 89
   utility value of 77
Weitzman’s welfare theorem 3–4, 5, 9, 30–32, 34–42
cost–benefit rules 42–3
genuine saving 44–6
real NNP growth and welfare 44
relationship between GCNNP growth and genuine saving 46–7
welfare
   analysis 130
   approximation of 24–5
   overestimation of 61
welfare measurement
   money metrics/utility metrics 30–32
   Nash feedback-loop games/
      Stackelberg games 134–9
   Ramsey growth model 13–15
   under distortionary taxes 71–85
welfare measures
   biases of 130
   decomposition of 126–8
   estimated contributions of 132
white noise 153, 159
Wiener process 149
willingness-to-pay 25, 48, 55, 58, 67, 84, 106–7, 109–11
Young’s theorem 52