In the earlier times the profession was trying to focus on empirical discipline. They felt that if you didn’t have so many equilibria, you would have more discipline with your analysis. So I had to hide any multiple equilibria in footnotes where future researchers could find them but where referees were less likely to see them. (William A. [“Buz’] Brock in Colander, Holt, and Rosser, 2004, p. 162)

What is complexity research about?
Complexity research is about complexity in various disciplines. It can be argued that ultimately this is a transdisciplinary view of the world, but it gets applied in specific disciplines. This book will focus on its applications in economics in particular, which means that we need to know what we mean by complexity, itself a complex issue, and one that is at the heart of several chapters in the first part of this book.

Let us posit three different levels of complexity. The lowest level, labeled “small tent” complexity by Rosser (1999), and also as “Santa Fe complexity,” involves a focus on interacting heterogeneous agents. In a widely noted formulation, Arthur, Durlauf, and Lane (1997) provided a list of six characteristics that this lowest level tends to exhibit: 1) dispersed interaction among heterogeneous agents acting on each other locally in some space, 2) no global controller that can exploit all opportunities or interactions, despite the possibility of some weak global interactions, 3) cross-cutting hierarchical organization with tangled relations, 4) continual adaptation and learning by evolving agents, 5) perpetual novelty as new markets, technologies, behaviors, and institutions create new niches in the ecology of the system, and 6) out-of-equilibrium dynamics with either zero or many equilibria existing, and the system unlikely to be near a global optimum. This then is an imperfect world of bounded rationality and unexpected events and processes.

This lowest level of complexity can be viewed as the central workhorse and focus of much of complexity research that has gone on and continues to go on, with various examples and discussions of this view in several of the chapters in this book. Although it had existed previously in such outposts as Brussels and Stuttgart, it was at Santa Fe that it achieved its breakthrough of recognition and focus. In economics this was symbolized by three volumes out of the Santa Fe Institute, appearing every nine years
Handbook of research on complexity

(Anderson, Arrow, and Pines, 1988; Arthur, Durlauf, and Lane, 1997; Blume and Durlauf, 2006). These saw the development of this concept of complexity from an initial revolutionary phase through an expanded set of applications to a consolidation of the approach,1 especially as the editors of the final volume argued the concept of complexity had been reconciled with neoclassical economics. Rosser (2008) argued that this represented the selection of papers in the volume and the avoiding of more controversial material and researchers, although it must be admitted that ideas considered heretical 20 years ago have come to be viewed as more acceptable today, if not completely so.

The next level of complexity up is what Rosser (1999) labeled “big tent” complexity, which was defined following Day (1994) in dynamic terms, as processes that for endogenous reasons fail to converge to either a point, a limit cycle, or a simple expansion or contraction. This definition will be discussed more in the third chapter of this book, but suffice it for now to note that it included a set of other views based on nonlinear dynamics that had historically preceded the appearance of this later interacting agents complexity, although they were not called complexity (and some would argue that they are not actually complexity). These were the first three of what Horgan (1997) derisively labeled “the four Cs,” the fourth being complexity. These first three were cybernetics, catastrophe theory,2 and chaos theory, each of which was seen as having been an intellectual fad that experienced a bubble and then deservedly crashed. In Rosser (2004) these were represented by important past papers applying them to a wide variety of fields of economics.3 Rosser argued that they grew out of each other, reaching a critical mass of intellectual weight in the most recent period. It may be this accumulation of intellectual weight to the point where most observers must now credit the complexity concept with a degree of seriousness that may justify the remarks of Blume and Durlauf (2006) regarding the acceptance of the idea, or some parts of it anyway, by neoclassical economics.

Finally, the highest level of complexity I shall now call here for the first time, meta-complexity. This is the level at which Seth Lloyd has long been located, famous for having gathered at least 45 different definitions of complexity (Horgan, 1997, Ch. 11). The dynamic definition noted above, which contains the four Cs, is but a small part of this, even if it is arguably the most widely used perspective in economics. But the movement of research has more recently moved into some of these other definitions and perspectives. An important alternative is the computational perspective, arguably first introduced into economics by the late Peter Albin (1982). Chapter 4 of this volume focuses on this approach to complexity. At the cutting edge of complexity research these alternative views of complexity have become increasingly important, and the computational view is deeply
linked to the widely used algorithmic perspective in broader, transdisciplinary complexity. However, it must be recognized that as one moves to the level of meta-complexity, the definition of what it is becomes much less distinct. Nevertheless, this book presents perspectives at this broader and higher level of current cutting-edge research in complexity in various areas of economics.

Where did complexity come from?
The question of where the idea of complexity came from has a simple answer: it came from mathematics. In this regard, Weintraub (2002) has argued that revolutions in the history of economics have reflected revolutions in the history of mathematics. So, as Mirowski (1989) has argued, late 19th-century neoclassical economics arose from the mathematics used in the physics of the mid-19th century. The formalistic and axiomatic approach associated with the proofs of the existence of general equilibrium and early game theory was associated with the ultra-formalism of the Bourbakist school of mathematics in France, out of which milieu Gérard Debreu came. The more recent relaxation of this axiomatism and the move to greater use of computer simulations that has been integral to much recent complexity research reflected trends in mathematics as well. In this regard, mathematical developments are the link between related developments occurring across many disciplines.

Thus, just as Buz Brock complained about how he used to have to hide his multiple equilibria in footnotes so that referees would not notice them, so it is that the mathematical foundations of modern complexity theory arose out of the deep past, with many of the discoverers shying away from the “monsters” they had discovered, preferring an orderly universe without such complications. A supreme example of this was Poincaré (1899), who arguably invented the qualitative theory of differential equations and bifurcation theory. From these he would observe the possibility of endogenously irregular dynamics, later known to be chaotic, which he rejected and shied away from. But the origins run deeper, even as they were often largely ignored.

Thus, Rössler (1997, Ch. 1) has argued that one can find the idea of self-similarity as it appears in fractal geometry in the work of the pre-Socratic philosopher, Anaxagoras. This may be a stretch, but Arnol’d (1992) argues that Poincaré was preceded in his understanding of bifurcation theory by Huygens in 1654, who studied the stability of cusp points in caustics and wave fronts. Mandelbrot (1983, p. 4) pinpoints a crucial discovery by Weierstrass in 1872 of a function that is continuous but non-differentiable everywhere. In 1879 Cayley would study path dependence in the form of the question of when would a simple cubic equation converge to which of
its three roots. In 1883 Cantor would discover transfinite sets and with his famous Cantor set, the possibility of sets of zero measure that nevertheless were at the level of the continuum and exhibited fractal dimensionality. In 1892 Lyapounov would initiate the study of the stability of dynamical systems, and in the 20th century, the Russian School (Andronov, 1928; Kolmogorov, 1941; Sharkovsky, 1964) would follow this to develop much of the apparatus of modern complexity mathematics. Electrical engineers would discover in reality actual complex dynamics, such as chaos (van der Pol and van der Mark, 1927). It would only be later that these developments would find their manifestation in economic research, although again there would be pioneers ahead of their time with little recognition in their day (Kalecki, 1935; Goodwin, 1951; Schelling, 1971).4

In any case, just as in economics there would long be resistance to the newer approach, so in mathematics it first took a long time for these “monsters” to be accepted as house pets that are really quite ordinary, arguably more ordinary than the perfected mathematical objects of ancient Euclidean geometry. Thus, as Benoît Mandelbrot (1983, p. 1) has put it: “Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.”

What is in this book?
Three further chapters occupy this opening Conceptual Overviews part of this book. The first of these is by an important figure from the Santa Fe complexity group, W. Brian Arthur. He emphasizes the themes noted in the characteristics listed above of the “small tent” complexity view, such as perpetual novelty and interacting agents. He notes also the important role of nonlinearity, particularly in the form of positive feedbacks in systems, which can both destabilize dynamics while bringing about the emergence of higher forms of order. Indeed, he poses an important and deep theme of complexity research, that order can come out of chaos. Complex systems can be self-ordering, against all apparent odds, as he demonstrates in his famous El Farol Bar example.

The next chapter is the first of a pair that can be seen as engaging in a debate over the proper definition of complexity. It is by J. Barkley Rosser, Jr. and poses and works through the contrast between the computational and dynamic views of complexity, ultimately arguing for the greater usefulness in economics of the dynamic view, while granting the greater rigor associated with the computational view.

The chapter following that one is by K. Vela Velupillai and presents an overview of the new computable economics view, which in turn rests on computational or algorithmic definitions of complexity. Velupillai argues that this developing perspective rests on deep roots in mathematical
logic and probability theory. He links these to the deep unavoidability of bounded rationality in the face of the multiform problems of computability and solvability of systems. He poses deeper questions about the axiomatic foundations of mathematics and the implications for economic theory of these problems and issues.

The second part of this volume focuses on Market Dynamics with two chapters. The first of these is by Cars H. Hommes, who further pursues the problem of bounded rationality and learning in complex markets. He considers problems of expectation formation, posing adaptive learning as a key to evolutionary selection and dynamics. The performance of different forecasting rules in systems with interacting heterogeneous agents is considered, and evidence from recent economic experiments is presented and reviewed. He notes the deep interaction between simple rules of thumb and more costly methods of forecasting, with dynamics often driven by this alternation as the former can destabilize a system and push it into more complex dynamical patterns, while the latter may induce periods of stability and simplicity.

The second chapter in this part is by Michael Kopel who reviews models of oligopoly dynamics. This is an important area in complexity research as well as more broadly in mathematical economics. After all, it was in studying duopoly that Cournot was the first to apply calculus to economic theory, and it would be to the same problems conceived dynamically that David Rand (1978) would be the first to consciously apply chaos theory to the study of economics. Kopel covers these matters but brings in the more advanced models of multidimensional chaos and multistability in which multiple basins of attraction in a system coexist. The complexities involved can include fractal basin boundaries and riddled basins. He provides both a historical perspective on these approaches as well as distinguishing how each apply differently to Cournot, Bertrand, and Stackelberg problems.

The third part of the book moves to Macroeconomic Issues, again with two chapters. The first by Alan Kirman deals with complexity and aggregation. Coming originally out of general equilibrium theory, Kirman was among the first to point out the depth of problems involved in attempting to aggregate so that an economy would be modeled by a representative agent. Thus, he has been a major herald and advocate of the heterogeneous agents approach to complexity research. In this chapter he reviews these original theoretical concerns and then moves on to a variety of applications of heterogeneous agent modeling from public goods games to the Schelling model of urban segregation to the evolution of market networks, drawing on his famous work on the Marseille fish market.

Richard H. Day then follows with a chapter on simplicity and macroeconomic complexity. He emphasizes the deep complexity concept
of the emergence of higher-order structures out of lower-level systems, applying this to problems of multilevel causality in economics from the micro through the meso to the macro. He considers various issues such as reducibility and discretization in the relationship between simplicity and complexity, eventually presenting some models of macroeconomic dynamics based on nonlinear foundations.

The fourth part deals with the newly emerging field of Econophysics and its application to financial markets. The first chapter in this part, and the longest in the entire volume, is by Thomas Lux on applications of statistical physics in finance and economics. He presents a broad overview of econophysics methods and approaches. While deeply sympathetic, he does not hesitate to point out problems with certain approaches, echoing the critiques he put forth with others quite recently (Gallegati et al., 2006). A centerpiece of this chapter is showing how when econophysicists fall into using orthodox economic theory as a foundation for their modeling, this can lead them into making drastically incorrect forecasts about financial market dynamics and futures.

The second of the two chapters is by Joseph L. McCauley, Kevin E. Bassler, and Gemunu L. Gunaratne, on the analysis of time series with nonstationary increments. They challenge a variety of established views in this chapter on time series analysis, drawing on ideas from physics. They argue that Hurst coefficients exhibit serious problems and they dispute the existence of kurtotic “fat tails” in certain market data. They pose the log return as a central conceptual focus to study Markov processes, scaling Ito processes, and financial market problems of liquidity, noise, and crashes of speculative bubbles. This chapter constitutes a challenge not only to orthodox economic theory and econometrics, but to what has emerged as more standard approaches used in econophysics as well.

The fifth part of the book focuses on International Economics. The first of the two chapters is by Frank H. Westerhoff, who surveys recent work on nonlinear modeling of exchange rate dynamics, which have long been notorious for being the most difficult of economic variables to model or forecast. A major thrust of this chapter is on the interaction between fundamentalists and chartists, or technical traders, in the foreign exchange markets, and how heterogeneous agents can move back and forth between strategies. While generally, technical traders tend to destabilize and complexify dynamics, they can also serve to some degree as stabilizers in these markets. The emphasis on learning and adaptation links this chapter to some of the others in the book and shows how the extreme volatility and unpredictability in these markets has made those studying them more open than many to considering the use of models using complexity approaches.
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The second chapter in this part is by Hans-Peter Brunner and Peter Allen, who consider complex systems modeling and international development. They focus on autocatalytic processes in developing economies and how a complexity perspective alters the approach to policy. Rather than imposed top-down interventions, the concept is that interventions must cohere well with the networks and structures of the socioeconomic system. The emphasis is on self-organization of these processes as stimulated by careful policy interventions. It presents an example of the approach using an input–output model of the poor state of Orissa in northeastern India, with its trade relations with other states and nearby countries, considering the possible outcomes from different levels of transportation infrastructure investment.

Part VI of this volume is on Evolutionary and Ecological-Environmental Economics. The first chapter is in the vein of evolutionary game theory by Herbert Gintis, Ross Cressman, and Thijs Ruijgrok on subgame perfection in evolutionary dynamics with recurrent perturbations. Under conditions of perturbation in finite noncooperative games with unique subgame perfect equilibria, they show how often there will not be convergence to this equilibrium; indeed that convergence may go quite far away from it. This fits in well with the theme of out-of-equilibrium dynamics in complex systems.

The second chapter by Rosser is on complex dynamics in ecologic-economic systems. This considers the peculiar complexities that can arise from the interaction of biology and economics in such coupled systems with their inevitable nonlinearities. Models of fisheries and forests are studied, as are models of climate change dynamics. The problem of hierarchy and broader evolutionary dynamics are also looked at.

The final part of the book moves on to broader historical perspectives of economic thought. The first chapter is by Roger Koppl and considers complexity and Austrian economics. Portions of his discussion echo the earlier chapter by Velupillai with a concern for the limits of computability and the implications for bounded rationality that he argues the Austrian perspective of Hayek and others implies for economic analysis. He poses the key to the links between complexity and Austrian economics to be “BRICE,” bounded rationality, rule following, institutions, cognition, and evolution. He points out in particular the role of Hayek as an early, conscious complexity theorist.

The final chapter in the volume is by David Colander on complexity and the history of economic thought. After considering alternative visions of economics as the complexity perspective contrasted with a more conventional or orthodox view, including a consideration of the boundaries of economics and how these might change as a result of complexity research,
he engages in a re-evaluation of past thinkers in economics in regard to how they fit in with the emerging complexity perspective. Among those moving up are Adam Smith, John Stuart Mill, Karl Marx, Alfred Marshall, and Friedrich Hayek. Those moving down include David Ricardo and Léon Walras. Those neutrally affected include both Thomas Robert Malthus and John Maynard Keynes, although each of them is partly moving up and partly moving down. He concludes by elevating both Charles Babbage and John von Neumann, whom he sees as important precursors of the later development of complexity research more generally.

Notes
1. Curiously the one person to appear in all three of these volumes was William Brock, quoted at the beginning of this Introduction.
2. Catastrophe theory in particular came in for much ridicule from Horgan and others. See Rosser (2007) for further discussion.
3. The least represented was the first, although it has arguably resurfaced most clearly in the newer heterogeneous agents complexity. It had two branches: cybernetics proper, founded by Norbert Wiener (1961), and system dynamics founded by Jay W. Forrester (1961). See Richardson (1991) for a discussion of the relationship between these two.
4. See Rosser (2000, Ch. 2) for a fuller discussion of the mathematical underpinnings of these developments.

References


