1 An introduction to the theory of social accounting

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1 BACKGROUND

At least since the 1970s, much research effort has been devoted to the use and design of national accounts. One of the basic ideas behind this work has been to provide a coherent framework for measuring national and/or global welfare in a dynamic economy as well as understanding how the current system of national accounts ought to be modified with this particular objective in mind. A suitable name for this research area is 'social accounting': according to *The New Palgrave Dictionary of Economics*, this refers to 'the body of data that portrays a nation's economic activity in terms of output produced and incomes created, the stocks of capital goods and other inputs required, and the financial pathways and instruments used'. Herein lies also the task of measuring the social value of this economic activity, which is where the welfare-economic perspective comes in. More specifically, measuring the social value of economic activity – whether this activity refers to a nation or a supranational community – requires a welfare economic theory of social accounting, and such a theory has gradually evolved (and is still evolving). The purpose of this introductory chapter is to briefly discuss some of the main insights that we believe that this theory has produced.[1] We will do so by focusing on three interrelated issues: (i) principles for measuring welfare in a community at a given point in time, (ii) cost–benefit rules for measuring welfare change and (iii) principles for measuring sustainability, which are the three main topics addressed by this Handbook. Our purpose here is to present a starting point for the analyses carried out in later chapters.

The outline of the present chapter is as follows. In Section 2, we discuss the problem of measuring welfare at a given point in time in a dynamic economy and, in particular, the relationship between welfare and the Hamiltonian. Section 3 deals with cost–benefit analysis – again in the context of a dynamic economy – whereas Section 4 deals with sustainability. We end the chapter by presenting an outline of the Handbook as well as briefly discussing each of the remaining chapters.

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2 WELFARE MEASUREMENT AND THE HAMILTONIAN

What does a 'national welfare measure' look like in a dynamic economy, and can we use entities from national accounts to calculate a static equivalent to this welfare measure? Most of the early welfare measures were wealth-like concepts such as the present value of future utility or consumption; see for example Samuelson (1961). However, although such a measure is accurate – as long as the future utility for society is measured correctly – it is not very practical. This is so for an obvious reason: it is forward looking and would, therefore, necessitate predictions of all welfare-relevant entities far into the future. Much would be gained if it were possible to construct a static equivalent to future utility. The comprehensive net national product, to be referred to as 'comprehensive NNP' for short, serves this purpose. The comprehensive NNP is an extension of the conventional NNP concept, where the extension captures *all* aspects of consumption and capital formation that are relevant for society. A comprehensive concept of consumption should reflect consumer preferences and not be restricted to conventional goods and services; it is also likely to include other 'utilities' such as leisure and environmental quality (entities which are not part of the conventional NNP). Similarly, a comprehensive measure of net investment should include all capital formation undertaken by society and not merely changes in the stock of physical capital. Other stocks of importance for production and/or utility are natural resource stocks, stocks that represent (or influence) environmental quality, and the stock of human capital. The net changes in these stocks, measured over a period of time, also qualify as capital formation. Depending on focus, the comprehensive NNP is sometimes also referred to as the 'green NNP', at least if applied to economies where consumption and/or capital aspects of the natural environment are important parts of the economic system. We shall here use the concepts of comprehensive and green NNP synonymously.

Weitzman (1976) was first to show that comprehensive NNP, if measured in terms of utility, constitutes an exact welfare measure in a dynamic economy. In technical terms, the result derived by Weitzman implies that the current value Hamiltonian of the underlying optimal growth problem is proportional to the present value of future utility facing the representative consumer. This Hamiltonian-based welfare measure is, therefore, a static equivalent – or annuity equivalent – to future utility, which explains why it is so frequently referred to in the literature. The current value Hamiltonian is, in turn, often interpreted as the comprehensive NNP, measured in units of utility, as it represents the utility value of current

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consumption (broadly defined to capture all goods and services that give rise to instantaneous utility) plus the utility value of all current investments (which include changes in all capital stocks of relevance for society). Although Weitzman himself did not use the term 'comprehensive NNP' in his 1976 paper, the NNP concept should be interpreted in a broader sense than the conventional NNP: Weitzman wrote that, in addition to physical, man-made capital, 'pools of exhaustible natural resources ought to qualify as capital, and so should stocks of knowledge resulting from learning or research activities'. This view of NNP has inspired much of the subsequent research on welfare measurement, where different aspects of capital formation have been addressed.[2]

As we will argue later, however, the welfare interpretation of the current value Hamiltonian relies on a set of assumptions, which appears to be somewhat restrictive. To be more specific, the Hamiltonian-based welfare measure assumes a stationary technology and an optimal resource allocation. The first assumption rules out disembodied technological change, whereas the second rules out typical market failures such as uninternalized externalities or involuntary unemployment. If either of these two assumptions is relaxed, the welfare measure will contain forward-looking terms (in addition to the current value Hamiltonian), which cannot be estimated solely by using information that is part of the current value Hamiltonian, that is, the Hamiltonian-based indicator is no longer an exact welfare measure. This suggests that market imperfections may undermine the welfare economic foundation for comprehensive NNP (if based on the Hamiltonian concept). For obvious reasons, it also means that practical applications of social accounting are difficult to carry out, at least if the welfare economic foundation is to be taken seriously.

2.1 A Dynamic Model with Stock Pollution

In this subsection, we present a model developed by Brock (1977), where production releases emissions, and where the stock of pollution – accumulated via emissions – gives rise to a consumption externality. We have chosen this particular model primarily for two reasons. First, the model is particularly suited for studying environmental aspects of social accounting, allowing us to connect with a major theme in earlier literature, namely, the welfare-economic foundations for 'green national accounts'. Second, by introducing a market failure, we are also able to distinguish between a first-best welfare measure and a welfare measure applicable in an imperfect market economy. The analyses carried out in subsections 2.2 and 2.3 below are largely based on Aronsson and Löfgren (1999a). To shorten the presentation as much as possible, we only consider

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utility-based welfare measures here; as our purpose is to address principles (not the step from theory to application), this implies no loss of generality by comparison with money-metrics based welfare measures.

We consider an economy where the consumers are identical and have infinite planning horizons.[3] We also follow the convention in much of the earlier literature on social accounting of disregarding population growth and normalizing the population to equal one. The instantaneous utility function at time t is written as

$$u(t) = u(c(t), x(t)) \tag{1}$$

where c is private consumption and x the stock of pollution. The consumer is assumed to supply one unit of labor inelastically at each instant. This simplification is justified here because endogenous labor supply behavior adds nothing essential to the analysis carried out below. We assume that the function $u(\cdot)$ is increasing in c, decreasing in x and strictly concave.

Turning to production, we assume that identical competitive firms, whose number is normalized to one, produce a homogeneous good by using labor (normalized to one and suppressed), physical capital and energy. The technology is stationary, and the production function is given by

$$y(t) = f(k(t), g(t)). \tag{2}$$

In equation (2), y denotes net output, meaning that depreciation has been accounted for, k the stock of physical capital and g energy input. We assume that the function $f(\cdot)$ is increasing in each argument and strictly concave.

The accumulation of pollution is governed by the following differential equation:

$$\dot{x}(t) = g(t) - \gamma x(t) \tag{3}$$

where $\gamma \in (0, 1)$ reflects the assimilative capacity of the environment. To connect emissions to energy input in a simple way, we assume (with little loss of generality) that the emissions equal the input of energy.

Finally, the accumulation of physical capital obeys the resource constraint

$$\dot{k}(t) = f(k(t), g(t)) - c(t). \tag{4}$$

Equation (4) means that net output is used for private consumption and net investment.

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2.2 First-Best Social Optimum and the Hamiltonian-Based Welfare Measure

To derive the first-best social optimum, it is convenient to assume that the resource allocation is decided upon by a social planner, whose objective coincides with the utility function facing the representative consumer. The decision-problem facing the social planner can be written as

$$\underset{c(t),\, g(t)}{Max} \int_0^\infty u(c(t), x(t))e^{-\theta t}dt \tag{5}$$

subject to

$$\dot{k}(t) = f(k(t), g(t)) - c(t) \tag{6}$$

$$\dot{x}(t) = g(t) - \gamma x(t) \tag{7}$$

as well as subject to the initial conditions $k(0) = k_0 > 0$ and $x(0) = x_0 > 0$, and the terminal conditions $\lim_{t\to\infty}k(t) \geq 0$ and $\lim_{t\to\infty}x(t) \geq 0$. Therefore, the decision-problem facing the social planner is here represented by a standard optimal control problem with two control variables, c and g, and two state variables, k and x. The parameter θ is the utility discount rate.

The present value Hamiltonian corresponding to the social planner's decision-problem becomes

$$H(t) = u(c(t), x(t))e^{-\theta t} + \lambda(t)\dot{k}(t) + \mu(t)\dot{x}(t). \tag{8}$$

In equation (8), λ and μ are costate variables associated with the physical capital stock and stock of pollution, respectively. In addition to equations (6) and (7), and in addition to the initial and terminal conditions, the necessary conditions for an interior social optimum include (where the time indicator has been suppressed for notational convenience)

$$\frac{\partial H}{\partial c} = u_c(c, x)e^{-\theta t} - \lambda = 0 \tag{9}$$

$$\frac{\partial H}{\partial g} = \lambda f_g(k, g) + \mu = 0 \tag{10}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial k} = -\lambda f_k(k, g) \tag{11}$$

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$$\dot{\mu} = -\frac{\partial H}{\partial x} = -u_x(c, x)e^{-\theta t} + \mu\gamma \tag{12}$$

where a subscript attached to the utility or production function denotes a partial derivative. Equations (9) and (10) are standard efficiency conditions for the control variables, c and g, whereas equations (11) and (12) are the equations of motion for the costate variables, that is, they show how the (utility-based) shadow prices of physical capital and pollution evolve over time along the optimal path. Note also that the costate variable $\mu(t)$ is the appropriate measure of social marginal cost of releasing emissions at time t (measured in units of utility), which the social planner weighs against the marginal benefit of higher output due to increased release of emissions, $\lambda(t)f_g(k(t), g(t))$. Let $\{c^*(t), g^*(t), k^*(t), x^*(t), \lambda^*(t), \mu^*(t)\}_0^\infty$ denote the resource allocation that obeys the optimality conditions presented above, where the superindex * denotes 'first-best optimum'; it represents the best possible outcome given the preferences and constraints described above.

For this economy, one can show that the Hamiltonian constitutes an exact welfare measure. By totally differentiating the present value Hamiltonian with respect to time (assuming differentiability) and using the necessary conditions given by equations (9)–(12), we have (suppressing the time indicator once again)

$$\begin{aligned}\frac{dH^*}{dt} = -\theta u(c^*, x^*)e^{-\theta t} + \frac{\partial H^*}{\partial c}\frac{dc^*}{dt} + \frac{\partial H^*}{\partial g}\frac{dg^*}{dt} + \frac{\partial H^*}{\partial k}\frac{dk^*}{dt} + \frac{\partial H^*}{\partial x}\frac{dx^*}{dt} \\ + \frac{\partial H^*}{\partial \lambda}\frac{d\lambda^*}{dt} + \frac{\partial H^*}{\partial \mu}\frac{d\mu^*}{dt} = -\theta u(c^*, x^*)e^{-\theta t}.\end{aligned} \tag{13}$$

In equation (13), H^* denotes the present value Hamiltonian evaluated in the first-best optimum. The expression after the second equality in equation (13) is a direct consequence of the dynamic envelope theorem: all indirect effects of time via control, state and costate variables vanish as a consequence of optimization.[4] Therefore, only the direct effect of time, $-\theta u(c^*, x^*)\exp(-\theta t) \equiv \partial H^*/\partial t$, remains in equation (13), which is due to the explicit time dependence of the utility discount factor. By solving equation (13) subject to the transversality condition $\lim_{T\to\infty}H^*(T) = 0$,[5] and transforming the solution to current value, that is, multiplying by $e^{\theta t}$, we obtain (where the superindex c stands for 'current value')

$$\theta V^*(t) = H^{c*}(t) \tag{14}$$

in which

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$$V^*(t) = \int_t^\infty u(c^*(s), x^*(s))e^{-\theta(s-t)}ds \tag{15}$$

is the optimal value function at time t (that is, the present value of future utility facing the representative consumer at time t under the optimal resource allocation), and $H^c(t) = H(t)e^{\theta t}$ is the current value Hamiltonian at time t.

Equation (14) is Weitzman's (1976) welfare measure applied to the model discussed here. It means that the present value of future utility at time t is proportional to the current value Hamiltonian at time t, where the utility discount rate constitutes the factor of proportionality. An important implication of equation (14) is that welfare at time t can be measured solely by using information referring to time t, although the welfare concept itself is fundamentally intertemporal. In other words, the Hamiltonian constitutes a static equivalent to future utility. The intuition is that, in a first-best resource allocation, the costate variables accurately reflect the future welfare consequences of the actions taken today. This will be described more thoroughly below. Furthermore, possible welfare contributions of technological change are ruled out by the stationary technology assumption made above.[6] Therefore, all sources of welfare are contained in – and accurately measured by – the current value Hamiltonian.

The current value Hamiltonian on the right-hand side of equation (14) can be written as

$$H^{c*}(t) = u(c^*(t), x^*(t)) + \lambda^{c*}(t)\dot{k}^*(t) + \mu^{c*}(t)\dot{x}^*(t) \tag{16}$$

where $\lambda^c(t) = \lambda(t)e^{\theta t}$ and $\mu^c(t) = \mu(t)e^{\theta t}$ are the costate variables at time t measured in current value. It has become common in earlier literature to interpret the current value Hamiltonian as a measure of comprehensive NNP in utility terms. The intuition is obvious from equation (16): the current value Hamiltonian reflects the instantaneous utility associated with the current consumption (the first term on the right-hand side) plus the utility value of the current net investments (the second and third terms). For the simple economy discussed here, the consumption concept contains two parts – goods and services, c, and pollution, x – whereas the net investments refer to the changes in the physical capital stock, $\dot{k}$, and the additions to the stock of pollution, $\dot{x}$.

To take the NNP interpretation one step further, let us first linearize the instantaneous utility function and then use equation (9) to rewrite the instantaneous utility function as follows:

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$$u(c, x) = \lambda^c[c + \rho x] + s$$

where $s = u(c, x) - \lambda^c c - u_x(c, x)x$ is the consumer surplus and $\rho = u_x(c, x)/u_c(c, x)$ the marginal rate of substitution between pollution and private consumption. We can now rewrite equation (14) as

$$\theta V^*(t) = \lambda^{c^*}(t)[c^*(t) + \dot{k}^*(t) + \rho^*(t)x^*(t) - \tau^*(t)\dot{x}^*(t)] + s^*(t). \tag{17}$$

In equation (17), $-\tau^*(t) = \mu^{c^*}(t)/\lambda^{c^*}(t) < 0$ is the real shadow price of additions to the stock of pollution at time t in the social optimum. As we will show below, $\tau^*(t)$ is also the (intertemporal analogue to the) Pigouvian emission tax that internalizes the consumption externality of pollution. Equation (17) shows that welfare – as represented by the present value of future utility facing the representative consumer – is proportional to the sum of two terms: the linearized current value Hamiltonian and the consumer surplus. The former is, in turn, defined as the real comprehensive NNP times the marginal utility value of capital. For the economy set out here, real comprehensive NNP contains four parts. The first two terms represent the conventional NNP, the third term measures the stock of pollution times the marginal value of this stock at time t (that is, the 'exchange-value' of the public bad at this particular time),[7] and the fourth term represents the marginal value of additions to the stock of pollution (the net investment aspect of the environment). Therefore, the third and fourth terms represent, in a sense, the additional information we would need in order to 'green' the national accounts. In general, therefore, the real comprehensive NNP does not constitute an exact real welfare measure due to the appearance of the consumer surplus in equation (17).[8] Note also that in the special – yet highly unrealistic – case where the instantaneous utility function is linear homogeneous, we have $s = 0$, meaning that the linearized current value Hamiltonian is proportional to the present value of future utility. The Cobb-Douglas utility function $u(c, x) = c^\alpha x^{1-\alpha}$, where $\alpha \in (0, 1)$, exemplifies such a utility function.

We mentioned above that, in the first-best optimum, the costate variables accurately measure the future welfare effects of current actions, that is, intertemporal social opportunity costs. To see this more clearly, let us solve the differential (12) for $\mu(t)$, subject to the transversality condition $\lim_{T\to\infty}\mu(T) = 0$. We have

$$\mu^*(t) = \int_t^\infty u_x(c^*(s), x^*(s))e^{-\theta s}e^{-\gamma(s-t)}ds < 0, \tag{18a}$$

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or, equivalently, in current value terms through multiplying by $e^{\theta t}$

$$\mu^{c*}(t) = \int_t^\infty u_x(c^*(s), x^*(s))e^{-(\theta+\gamma)(s-t)}ds < 0. \tag{18b}$$

Recall from equation (10) that the shadow price of pollution at time t constitutes the social marginal cost of releasing emissions at time t. In equations (18a) and (18b), we can see that this shadow price is forward looking: it measures the present utility value of the future increases in the stock of pollution that the release of emissions at time t gives rise to. Note also that γ, the rate of depreciation of pollution, appears as an extra discount factor. The intuition is, of course, that the higher the rate of depreciation, *ceteris paribus*, the less will be the effective increase in the future stock. If we were to measure the shadow price of pollution in real terms, instead of in units of utility as in equations (18a) and (18b), we would arrive at the (negative of the) Pigouvian tax mentioned above, that is, the tax that in a market economy would induce the firm to release the socially optimal level of emissions. Therefore, the Pigouvian tax,

$$\tau^*(t) = -\frac{\int_t^\infty u_x(c^*(s), x^*(s))e^{-\theta s}e^{-\gamma(s-t)}ds}{\lambda^*(t)} > 0, \tag{18c}$$

is interpretable in the same general way as the utility-based shadow price. We will return to the Pigouvian tax below, where we consider the welfare measurement problem in the context of a market economy (instead of an economy where the resource allocation is decided upon by a social planner).

Extension: Welfare measurement in a stochastic environment

The analysis carried out above assumes perfect certainty. Although convenient from an analytical point of view, this assumption is hardly realistic. However, the step towards uncertainty need not necessarily be very complicated. The Hamilton-Jacobi-Bellman equation (a stochastic partial differential equation obeyed by the optimal value function) from stochastic control theory can be used to derive a welfare measure which is an analogue to the deterministic welfare measure analysed above. In the special case with perfect certainty – and under the same assumption about the preferences and technology as those set out above – this technique reproduces the welfare measure presented in equation (14).

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Aronsson and Löfgren (1995) consider welfare measurement in a stochastic version of the Ramsey model, in which the rate of population growth follows a Brownian motion. They show that welfare – represented by the expected present value of future utility facing the representative consumer – is measured by a generalized current value Hamiltonian, where the generalization means that it also reflects the valuation of the risk associated with a given investment. If the individual is risk-averse (risk-loving), then this extra term contributes to lower (higher) welfare. As the additional term is proportional to the variance of the stochastic variable, it vanishes under perfect certainty, and we are back in the deterministic welfare analysis addressed above. Weitzman (1998) derives an analogous welfare measure for an economy where the rate of time preference (instead of the rate of population growth) follows a Brownian motion.[9]

2.3 Welfare Measurement in an Imperfectly Controlled Market Economy

In a decentralized economy, the resource allocation is not necessarily optimal from society's point of view. The model presented in subsection 2.1 contains a consumption externality, since the release of emissions by firms builds up a stock of pollution, which in turn directly affects the utility of the representative consumer. We will here address the implications for welfare measurement that will follow, if this externality has not become fully internalized.

The structure of the model is the same as before; the only difference is that the decisions about consumption, capital formation and production are here made by consumers and firms instead of by a social planner. The utility maximization problem facing the consumer is given by

$$\underset{c(t)}{Max}\int_0^\infty u(c(t), x(t))e^{-\theta t}dt \tag{19}$$

subject to the asset accumulation equation

$$\dot{k}(t) = \pi(t) + r(t)k(t) + w(t) + T(t) - c(t), \tag{20}$$

as well as subject to the initial condition $k(0) = k_0$, and a No-Ponzi Game (NPG) condition meaning that the present value of the asset (physical capital) is non-negative at the terminal point. The consumer supplies one unit of labor inelastically at each instant and earns labor income $w(t)$ as well as renting out capital at the market rate of interest $r(t)$ to the

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representative firm. The variables $\pi(t) \geq 0$ and $T(t) > 0$ represent possible profit income and a lump-sum transfer (to be defined below), respectively. Note that the representative consumer treats the stock of pollution, the labor income, the interest rate, the profit income and the lump-sum transfer as exogenous.

The representative firm chooses capital, $k(t)$, and emissions, $g(t)$, to maximize profit at each point in time. Its objective function at time t can then be written as

$$\pi(t) = f(k(t), g(t)) - w(t) - r(t)k(t) - \tau(t)g(t). \tag{21}$$

In equation (21), $\tau(t)$ is a tax per unit of emissions paid by the firm at time t. Finally, budget balance for the public sector implies that $\tau(t)g(t) = T(t)$ for all t.

If we combine the first-order conditions for the consumer and the firm, the following conditions are among those obeyed by the decentralized equilibrium (suppressing the time indicator for notational convenience);

$$u_c(c, x)e^{-\theta t} - \lambda = 0 \tag{22}$$

$$f_g(k, g) - \tau = 0 \tag{23}$$

$$\dot{\lambda} = -\lambda f_k(k, g). \tag{24}$$

There are two principal differences between the necessary conditions characterizing the decentralized economy and the first-best optimal resource allocation. First, the emission tax is not necessarily the Pigouvian tax described in the previous subsection. Second, the stock of pollution is not an endogenous state variable in the decentralized economy; it is, instead, a side effect of the behavior of the firm and treated as exogenous by the consumer.

Let $\tau^0(t)$ for $t \in [0, \infty)$ represent the emission tax path decided upon by the policy maker (to be discussed more thoroughly below), where the superindex 0 denotes 'imperfectly controlled market economy', so as to distinguish it from the first-best optimum analyzed in subsection 2.2. Suppose that this emission tax path gives rise to the resource allocation

$$\{c^0(t), g^0(t), k^0(t), x^0(t), \lambda^0(t)\}_0^\infty.$$

Note that the first-order conditions presented in equations (22)–(24) look as if they are derived from the following present value pseudo-Hamiltonian (where the subindex p stands for 'pseudo'):

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$$H_p^0(t) = u(c^0(t), x^0(t))e^{-\theta t} + \lambda^0(t)\dot{k}^0(t) - \lambda^0(t)\tau^0(t)\dot{x}^0(t). \tag{25}$$

In equation (25), we can interpret $-\lambda^0(t)\tau^0(t)$ as an estimate of the shadow price of pollution at time t. If the resource allocation were first best, and thus $\tau^0(t) = \tau^*(t) = -\mu^*(t)/\lambda^*(t)$ would be the Pigouvian emission tax at time t, then $c^0(t) = c^*(t)$, $g^0(t) = g^*(t)$, $k^0(t) = k^*(t)$ and $x^0(t) = x^*(t)$. In this case, equation (25) would be the present value Hamiltonian, and it would constitute an exact annuity equivalent to future utility for the reasons presented in subsection 2.2.

Although public policy aims at correcting for the environmental externality, the emission tax does not necessarily need to be an accurate estimate of the marginal social value of an increase in the stock of pollution at each instant; after all, this is the essence of the phrase 'imperfectly controlled market economy'. To operationalize this idea, we follow Aronsson and Löfgren (1999a) by assuming that the actual emission tax reflects a biased estimate of the marginal utility of pollution in the following sense:

$$\tau^0(t) = -\frac{\int_t^\infty (u_x(c^0(s), x^0(s)) + \beta(s))e^{-\theta s}e^{-\gamma(s-t)}ds}{\lambda^0(t)} \tag{26}$$

where the time-varying variable β represents the instantaneous bias in the estimate of the marginal utility of pollution. Therefore, if $\beta(t) = 0$ for all t, then equation (25) takes the same form as the Pigouvian tax. Similarly, if $\beta(t) = -u_x(c(t), x(t))$ for all t, the resource allocation represents an uncontrolled market economy where $\tau^0(t) = 0$ for all t. In other words, equation (26) is general enough to encompass both the first-best social optimum and the uncontrolled market economy as special cases.

As before, welfare is defined as the present value of future utility facing the representative consumer, that is, the optimal value function. In the decentralized, and possibly imperfect market economy, the optimal value function at time t can be written as follows:

$$V^0(t) = \int_t^\infty u(c^0(s), x^0(s))e^{-\theta(s-t)}ds. \tag{27}$$

What is the relationship between the optimal value function and pseudo-Hamiltonian defined in equation (25)? If we differentiate equation (25) totally with respect to time and use the necessary conditions in equations

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(22), (23) and (24), we have (suppressing the time indicator for notational convenience)

$$\frac{dH_p^0}{dt} = -\theta u(c^0, x^0)e^{-\theta t} + [u_x^0 e^{-\theta t} + \lambda^0\tau^0(f_k^0 + \gamma) - \lambda^0\dot{\tau}^0]\dot{x}^0 \tag{28}$$

where we have used the short notations $u_x^0 = u_x(c^0, x^0)$ and $f_k^0 = f_k(k^0, g^0)$. Now, observe that the time derivative of the emission tax in equation (26) can be written as

$$\dot{\tau}^0(t) = \frac{[u_x^0(t) + \beta(t)]e^{-\theta t}}{\lambda^0(t)} + \tau^0(t)(f_k^0(t) + \gamma). \tag{29}$$

Substituting equation (29) into equation (28) gives

$$\frac{dH_p^0(t)}{dt} = -\theta u(c^0(t), x^0(t))e^{-\theta t} - \beta(t)e^{-\theta t}\dot{x}^0(t). \tag{30}$$

Therefore, the non-autonomous time dependence arises from two sources here: the utility discount factor (as before) and the bias component of the emission tax. By solving equation (30) subject to the transversality condition $\lim_{T\to\infty} H_p^0(T) = 0$ and, finally, transforming the solution to current value, we obtain

$$\theta V^0(t) = H_p^{c^0}(t) - \int_t^\infty \beta(s)e^{-\theta(s-t)}\dot{x}^0(s)\,ds. \tag{31}$$

The first term on the right-hand side of equation (31) is the pseudo-Hamiltonian measured in current value, with the same interpretation as its counterpart in the first best, whereas the second term is the present value of future biases, that is, the present value of the uninternalized marginal externality. This component arises for one single reason: the actual emission tax is based on a biased estimate of the marginal utility of pollution, meaning that the resource allocation is not optimal from society's point of view. In the first best, where $\beta(t) = 0$ for all t, the second term on the right-hand side vanishes, and equation (31) will reproduce the first-best welfare measure in equation (14). The polar case of an 'uncontrolled market economy' corresponds in our model to $\beta(t) = -u_x(c^0(t), x^0(t))$ for all t, meaning that the emission tax is equal to zero along the whole general equilibrium path. In the uncontrolled market economy, therefore, equation (31) changes to read

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$$\theta V^0(t) = H_p^{c^0}(t) + \int_t^{\infty} u_x(c^0(s), x^0(s)) e^{-\theta(s-t)} \dot{x}^0(s) ds.$$

Two things are worth noticing in equation (31). First, if the resource allocation is not optimal, the Hamiltonian at time t no longer contains all the information needed to measure welfare at time t. Second, the final term on the right-hand side of equation (31) is forward looking. In other words, we can no longer measure welfare at time t solely by using information referring to time t. The intuition is, of course, that the release of emissions in the present gives rise to increased pollution in the entire future, *ceteris paribus*. Therefore, if the future welfare consequences of released emissions at time t are measured incorrectly, we must adjust the welfare measure to reflect this whole path of future biases. This is precisely what happens here, which explains why the Hamiltonian does not constitute an exact welfare measure.

3 COST–BENEFIT ANALYSIS

The concept of welfare measurement need not only refer to the welfare level; it may also refer to changes in welfare following, for example, policy projects or other parametric changes in the economic system. This leads naturally to the principles of cost–benefit analysis in dynamic economies. It should be emphasized that although projects may be small or temporary, they are still likely to have intertemporal consequences. Therefore, the study of cost–benefit analysis in dynamic economies generates insights that are not easily gained in static models.[10] We will give two examples here; one refers to the welfare effect of a policy project in a first-best type of economy, where the resource allocation is defined conditional on the policy parameter in question, and the other refers to the welfare effect of increased emission taxation in an imperfectly controlled market economy.

Example 1: A parameter change in the first best

Consider once again the first-best resource allocation presented in subsection 2.2, with the modification that the resource constraint is rewritten as follows:

$$\dot{k}(t) = f(k(t), g(t), \alpha) - I(\alpha) - c(t). \tag{32}$$

The parameter α measures the resources spent on a project – for example, R&D or a public infrastructure investment – that leads to increased output.

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We also assume that the intensity parameter α is associated with a social cost, represented by the function $I(\alpha)$, which is increasing and convex in α. Given that the economy has reached a first-best resource allocation defined conditional on α, we would like to measure the welfare effect of a small permanent increase in α. The present value Hamiltonian plays a crucial role in this welfare change measure; the direct effect on the Hamiltonian of a change in this policy parameter represents, in a sense, a measure of instantaneous welfare change. The total welfare change is then found by integration of instantaneous welfare changes over the planning period.[11]

To see this, let us write the optimal value function at time zero as follows:

$$V^*(0, \alpha) = \int_0^\infty u(c^*(t, \alpha), x^*(t, \alpha))e^{-\theta t}dt, \tag{33}$$

which emphasizes that the resource allocation is defined conditional on the parameter α. We show in the Appendix to this chapter that the welfare effect of a small change in α can be written as

$$\begin{aligned}\frac{\partial V^*(0, \alpha)}{\partial \alpha} &= \int_0^\infty \lambda^*(t, \alpha)[f_\alpha(k^*(t, \alpha), g^*(t, \alpha), \alpha) - I_\alpha(\alpha)]dt \\ &= \int_0^\infty \frac{\partial H^*(t, \alpha)}{\partial \alpha}dt. \end{aligned} \tag{34}$$

Here, $f_\alpha(\cdot)$ represents the direct effect on output of an increase in α with k and g held constant at their initial levels. The expression in square brackets after the first equality in equation (34) contains two components: the instantaneous direct marginal benefit and the instantaneous direct marginal cost of the project measured in utility terms. Note also that each difference between instantaneous marginal benefit and instantaneous marginal cost is discounted to present value via λ (the present value costate variable associated with the capital stock), and the instantaneous net benefits are then integrated over the whole planning period. The expression after the second equality emphasizes that all indirect effects of α vanish, as the initial resource allocation is optimal conditional on α; this is a consequence of the dynamic envelope theorem. Therefore, the welfare change can be measured by taking the partial derivative of the present value Hamiltonian with respect to α and then integrating over the planning period.

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Example 2: A small increase in the emission tax in the market economy
Consider once again the resource allocation addressed in subsection 2.3. Suppose now that the emission tax is increased from $\tau^0(t)$ to $\tau^0(t) + \vartheta$ for all t, where ϑ is a small positive constant. The additional tax revenue is redistributed lump-sum to the consumer. This policy is interpretable in terms of a small permanent increase in the emission tax. If the resource allocation obeys equations (22), (23) and (24), the optimal value function may be written as

$$V^0(0, \vartheta) = \int_0^\infty u(c^0(t, \vartheta), x^0(t, \vartheta))e^{-\theta t}dt \tag{35}$$

where the initial, that is, pre-reform equilibrium analyzed in subsection 2.3 means that $\vartheta = 0$. By applying the same technique as the one we used above, we can derive the cost–benefit rule that we are looking for. By evaluating this rule in the initial equilibrium where $\vartheta = 0$, we have

$$\frac{\partial V^0(0, \vartheta)}{\partial \vartheta} = \int_0^\infty \Bigg[u_x(c^0(t, \vartheta), x^0(t, \vartheta))e^{-\theta t}\frac{\partial x^0(t, \vartheta)}{\partial \vartheta} + \lambda^0(t, \vartheta)\tau^0(t)\frac{\partial g^0(t, \vartheta)}{\partial \vartheta}\Bigg]dt. \tag{36}$$

The first term within the square bracket on the right-hand side is the instantaneous marginal benefit of a reduction in the stock of pollution. This component contributes to higher welfare because $u_x(c^0, x^0) < 0$ and $\partial x^0(\cdot)/\partial\vartheta < 0$.[12] Similarly, the second term within the square bracket is the cost of higher taxation in terms of lost consumption (the higher emission tax leads to reduced output, *ceteris paribus*). This component is clearly negative since $\tau^0 > 0$ and $\partial g^0(\cdot)/\partial\vartheta < 0$. One can show that the welfare effect measured by equation (36) vanishes if the initial emission tax is Pigouvian, in which case the resource allocation is first best. One can also relate the welfare effect of increased emission taxation to the extent to which the actual pre-reform emission tax is a biased estimate of the Pigouvian emission tax (for further discussion, see Aronsson and Löfgren 1999a).

Finally, in the uncontrolled market economy, where the emission tax is equal to zero, equation (36) reduces to read

$$\frac{\partial V^0(0, \vartheta)}{\partial \vartheta} = \int_0^\infty u_x(c^0(t, \vartheta), x^0(t, \vartheta))e^{-\theta t}\frac{\partial x^0(t, \vartheta)}{\partial \vartheta}dt > 0. \tag{37}$$

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This neat – yet unsurprising – result means that it is always welfare improving to introduce a small emission tax in the uncontrolled market economy. The intuition is that the policy reform contains no first-order welfare cost in this case, while it gives rise to a first-order welfare benefit in terms of reduced pollution.

4 SUSTAINABILITY

Many of the basic ideas underlying the study of sustainability are thoroughly examined in later chapters. We will, therefore, limit the presentation here to a brief discussion of two interrelated issues: (i) genuine saving and (ii) conditions for non-declining instantaneous utility.

The World Commission on Environment and Development, often referred to as the Brundtland Commission, suggested in its 1987 report that 'sustainable development' ought to imply 'development that meets the need of the present without compromising the ability of future generations to meet their own needs'. One possible interpretation of this idea is that sustainable development requires that welfare is non-declining. This suggests, in turn, that the genuine saving, which is an exact measure of welfare change over a short time interval, also constitutes a *local* indicator of sustainable development.[13] Genuine saving is meant to imply the value of comprehensive net investments, possibly augmented with the marginal value of externalities and/or technological change depending on the functioning of the economic system. The focus on a short time interval also explains the above emphasis on the word 'local'.

To see that the genuine saving has this particular property, we shall once again use the model set out in subsection 2.1. We begin by differentiating the optimal value function with respect to time. Independently of whether the resource allocation is first best or an imperfectly controlled market economy, that is, independently of whether the calculation is based on equation (15) or equation (27) above, we obtain

$$\dot{V}(t) = -u(c(t), x(t)) + \theta V(t). \tag{38}$$

If the resource allocation is first best, we can use $\theta V^*(t) = H^{c*}(t)$ from equation (14) and substitute into equation (38). This gives the welfare-change measure

$$\dot{V}^*(t) = \lambda^{c*}(t)\dot{k}^*(t) + \mu^{c*}(t)\dot{x}^*(t). \tag{39a}$$

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In the imperfectly controlled market economy, an analogous mathematical operation gives

$$\dot{V}^0(t) = \lambda^{c^0}(t)\dot{k}^0(t) + \mu^{c^0}(t)\dot{x}^0(t) - \int_t^\infty \beta(s)e^{-\theta(s-t)}\dot{x}^0(s)ds \tag{39b}$$

where $\mu^{c^0}(t) = -\tau^0(t)\lambda^{c^0}(t)$. The right-hand side of equations (39a) and (39b) represent the genuine saving, measured in units of utility. Note that the final term on the right-hand side of equation (39b) is forward looking. The intuition is the same as that behind the forward-looking term in the welfare measure discussed in subsection 2.3: the incentives facing consumers and firms at time t (which govern the decisions made at that time) do not accurately reflect all social opportunity costs. Note also that, if we were to extend the model by allowing for disembodied technological change, yet another forward-looking term – the present value of marginal technological change – would enter the measure of genuine saving (see Aronsson, Johansson and Löfgren, 1997, p 106).[14]

It is important to emphasize once again that, if the genuine saving is used as an indicator of sustainability, it only constitutes a local indicator. For instance, the genuine saving says nothing of whether the instantaneous utility is increasing or decreasing along the future equilibrium path. This insight becomes particularly interesting if we reinterpret the model in terms of a continuum of generations (instead of in terms of a single individual with infinite time horizons). To see the argument more clearly, let $i^*(t)$ and $i^0(t)$ denote the measure of genuine saving on the right-hand side of equations (39a) and (39b), respectively. Integrating the final term on the right-hand side of equation (38) by parts (recalling the definition of the optimal value function in subsections 2.2 and 2.3), we can rewrite equations (39a) and (39b) as follows:

$$i(t) = \int_t^\infty \frac{du(c(s), x(s))}{ds}e^{-\theta(s-t)}ds = \int_t^\infty [u_c(s)\dot{c}(s) + u_x(s)\dot{x}(s)]e^{-\theta(s-t)}ds. \tag{40}$$

Therefore, independently of whether genuine saving is defined as in equation (39a) or equation (39b), it reflects a weighted average of changes in future instantaneous utilities. In other words, even if the genuine saving is positive, this does not mean that the instantaneous utilities are increasing along the general equilibrium path: only that a weighted average of future changes in instantaneous utilities is positive.[15] To ensure that the instantaneous utilities are non-declining at each instant along the equilibrium

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path, we would have to impose conditions on the genuine saving at each instant along this path. The reader is here referred to Hartwick (1977) and Dixit et al. (1980).

5 PLAN OF THE HANDBOOK

As we mentioned before, the main topics of the Handbook are: principles for measuring welfare in a community at a given point in time; cost–benefit rules for measuring the welfare change caused by policy (and other) projects; and principles for measuring sustainability. Each of the ten forthcoming chapters addresses one or several of these main topics and extends our knowledge in a variety of directions.

Welfare is typically measured in utility units, and the dominating bulk of earlier studies on social accounting have focused on utility-based welfare measures. Although this approach is theoretically convenient, it is less useful in practical applications simply because utility is unobservable. Therefore, it is important to find a way to transform a utility-based welfare measure into a corresponding money-metrics measure; an issue complicated by the fact that the marginal utility of income is not in general constant over time. In Chapter 2, Karl-Gustaf Löfgren develops a framework for money-metrics-based welfare measurement, showing how an ideal (benchmark independent) price index enables us to take the step from a utility-based welfare measure to a corresponding money-metrics-based measure. The results imply, among other things, that the comprehensive NNP augmented with the consumer surplus constitutes an exact money-metrics welfare measure in a first-best resource allocation. The analysis also shows how money-metrics-based welfare measures for imperfectly controlled market economies can be designed. Yet another issue that the chapter addresses is the relationship between welfare change and growth in real comprehensive NNP.

The traditional approach to social accounting in imperfect market economies has been to focus on the consequences of market failures (such as externalities or unemployment). However, modern literature on behavioral economics also recognizes the possibility that individuals may not be fully rational from the perspective of their own long-run interests. This idea is formalized by the concept of hyperbolic discounting: a mechanism of behavioral failure discussed in recent literature on paternalistic motives for public policy. In Chapter 3, written by Kenneth Backlund and Tomas Sjögren, hyperbolic discounting is integrated into the theory of social accounting. If not properly internalized, behavioral failures imply that the actual resource allocation may deviate from the one that a social planner

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(who acts on the basis of the consumers' long-run interests) would prefer. The results show that hyperbolic discounting leads to additional, and forward-looking terms in welfare measures. In general, therefore, self-control problems give rise to complications reminiscent of those associated with market failures. The chapter also analyzes, by means of numerical simulations, how effective paternalistic policies are for improving welfare.

Human beings typically face a variety of health risks, such as those associated with being in a particular environment. As these risks directly affect the expected utility, they are also relevant for welfare measurement. At the same time, when faced by risk, individuals are free to choose whether to invest resources to change the likelihood of future good and bad states of nature. Therefore, an important question is how such investments ought to be treated in social accounting. To answer this question, one must first understand the incentives underlying the investment behavior. In Chapter 4, Ram Ranjan and Jason F. Shogren develop a model economy where the representative consumer faces multiple risks of health breakdown, and where he/she can spend resources on self-protection and self-insurance. Among the results, the authors show that self-protection and self-insurance against a particular risk, when considered under multiple risks, can be either higher or lower than that arising under a single risk. In addition, the presence of multiple risks leads to discounting of those health risks that cause lower damage. From the perspective of practical application, measuring the marginal social value of accumulated risk becomes a major challenge for social accounting.

In all developed economies, the public sector plays a crucial role in resource allocation by providing public services and by income redistribution among individuals and groups. In fact, it is not uncommon that government outlays are in the neighborhood of 35–40 per cent of GDP. Furthermore, as the public revenue is raised by distortionary taxes, there is an additional cost of public funds that ought to be considered when measuring welfare. Chapter 5, written by Thomas Aronsson, concerns the treatment of distortionary taxation, public goods and income redistribution in the context of welfare measurement. The analysis is based on a second-best framework, in which the government raises revenue by using either a linear or non-linear income tax. Among other things, the chapter presents a second-best analogue to comprehensive NNP based on the Hamiltonian concept, and the results also show how the marginal cost of public funds affects the accounting price of a public good. In general, and in contrast to first-best welfare measures, a second-best analogue to real comprehensive NNP is not interpretable as an index comprising only aggregate variables; instead, it also reflects the distribution of private consumption and leisure among the consumers.

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Alongside the development of a theory of social accounting, a large empirical literature has also evolved. In addition, due to the increased awareness among policy makers that economic growth and environmental concerns ought to be considered simultaneously (instead of as two separate aspects of public policy), attempts have also been made to augment the conventional system for national accounts with important aspects of the natural environment, for example, the value of consumption of 'environmental goods or bads' and investments in different forms of natural capital. Chapter 6, written by Eva Samakovlis, addresses green accounting from a practical perspective, that is, how such accounts are produced in practice. As a number of countries have agreed to introduce green national accounts, a natural starting point here is the guidance provided by the *Handbook for integrated environmental and economic accounting* published by the United Nations et al. (2003), as well as how the environmental accounting programs of different countries relate to these guidelines. The chapter also overviews country-specific experiences in calculating comprehensive NNP (or parts thereof) as well as measurement of genuine saving. There is also discussion about some of the difficulties involved when attempting to construct welfare measures based on green national accounts.

Cost–benefit analysis is a tool for evaluating whether or not a public project is desirable. While the early literature often focused on capital investment projects, the focus has gradually shifted towards policy reform projects or projects relating to the natural environment. As such, part of the modern theory of cost–benefit analysis in dynamic economies has evolved parallel with theories referring to green national accounting and indicators of sustainability. In fact, one may argue that all these developments are to some extent different sides of the same underlying welfare economic theory. In Chapter 7, Chuan-Zhong Li reviews recent advances in the theory of dynamic cost–benefit analysis. The chapter presents and discusses a number of equivalent cost–benefit rules relating to small projects. It also explains the role of comprehensive NNP in cost–benefit analysis, which exemplifies the close connection between cost–benefit rules in dynamic economies and green accounting. The chapter ends with a discussion of the cost–benefit analysis of larger projects that involve price changes. By the result that the Hamiltonian (under ideal conditions) is an exact measure of future utility, the results show that the welfare change measure can be written in the same general way as in the corresponding static theory of cost–benefit analysis.

To estimate the costs of carbon dioxide emissions, we must be able to assess how the global surface temperature responds to the release of carbon dioxide into the atmosphere. Equilibrium climate sensitivity is a

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key stochastic parameter that converts relative changes in the concentration of atmospheric carbon dioxide into temperature change. It is defined as the global average surface warming following a doubling of carbon dioxide concentration. In Chapter 8, Martin Weitzman addresses the dynamics of climate sensitivity and, in particular, the fat upper tail of its distribution. He shows that the relevant posterior-predictive probability density function of a high-impact low-probability event has a built-in tendency to be fat tailed: the intuition is the difficulty of extrapolating extreme-impact tail behavior from finite data. The analysis also shows that the previous two-period result that fat-tailed climate sensitivity can have strong economic implications survives in a more complete dynamic specification. Therefore, fat tails in the distribution of climate sensitivity in combination with the possibly catastrophic effects of climate change might be of considerable importance for cost–benefit analysis and may even outweigh the influence of discounting.

Since the 1970s, the concept of sustainability has become increasingly important in economic analyses. A key question here is how to maintain a given flow of valuable products when its presence is threatened by various aspects of capital depletion. For instance, non-renewable capital stocks such as oil are inevitably depleted when used as inputs in production. A solution to this problem is given by the Hartwick rule: with some degree of substitutability between different types of capital, non-declining consumption can be maintained by keeping the combined capital in the economy intact over time, that is, the value of net investment – measured over all relevant capital stocks – must be non-negative at each point in time. In Chapter 9, John Hartwick surveys recent literature on sustainable per capita consumption programs. He focuses particular attention on two issues (or 'twists' as he calls them). The first is population growth, that is, how such growth affects the policies necessary to achieve non-declining consumption into the indefinite future, while the second is global warming, where current oil use causes temperature increase via the release of carbon dioxide emissions. The chapter also examines economies with a possibility of boundary-hitting collapse, as well as addressing the formal welfare economics of the notion of sustainability.

As we indicated above, there has been a parallel evolution of theories of social accounting – where a basic purpose has been to construct national (and global) welfare measures in dynamic economies – and theories underlying indicators of sustainability. This is not a coincidence, as concerns for intergenerational well-being are, in a sense, natural aspects of welfare in dynamic economies where future generations are affected by the actions taken today. An interesting question is whether or not measures of welfare improvement can also be used as indicators of sustainability.

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This is the starting point of Chapter 10, written by Geir Asheim. The welfare concept used here is total utilitarianism, where instantaneous well-being is given by the product of the population size and the instantaneous utility of per capita consumption, whereas concern for sustainability is based on non-declining utility of per capita consumption. This framework is then used to analyze whether a non-negative value of net investment (that is, genuine saving) implies sustainable development, and whether sustainable development implies a non-negative value of the net investment. Among the results, the author shows that welfare improvement – as measured by a positive value of net investment adjusted for population growth – is not a sufficient condition for non-declining utility of per capita consumption.

The insight that genuine saving under certain conditions is an exact measure of welfare change in a dynamic economy has inspired much recent discussion, and the World Bank now publishes numbers for genuine saving. However, as we mentioned above, genuine saving only constitutes a local indicator of sustainability (by measuring welfare improvement over a short time interval); a non-negative number for genuine saving does not imply that the consumption or utility is sustainable over a longer period. Chapter 11, written by Kirk Hamilton, discusses genuine saving in detail. In addition to a theoretical review, the chapter also contains information on genuine savings in different parts of the world as well as discussing results from empirical research on the relationship between genuine saving and future social welfare. A basic message is the central role that genuine saving can play for countries that aim at accelerating development, as it is both an indicator of welfare change and an indicator of unsustainable development (as a negative number for genuine saving implies that utility must eventually decline). As a consequence, genuine saving is likely to remain as a source of information of importance for public policy.

APPENDIX

To derive equation (34), we apply the approach to the dynamic envelope theorem suggested by Léonard (1987). Define the present value Hamiltonian

$$\overline{H}(t, \alpha) = u(c^*(t, \alpha), x^*(t, \alpha))e^{-\theta t} + \lambda(t)\dot{k}^*(t, \alpha) + \mu(t)\dot{x}^*(t, \alpha) \tag{A1}$$

for arbitrary and differentiable functions $\lambda(t)$ and $\mu(t)$. In equation (A1), the equations of motion for physical capital and pollution are given by

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$$\dot{k}(t, \alpha) = f(k(t, \alpha), g(t, \alpha), \alpha) - I(\alpha) - c(t, \alpha)$$

and

$$\dot{x}(t, \alpha) = g(t, \alpha) - \gamma x(t, \alpha)$$

in equilibrium. With the exception that the functions $\lambda(t)$ and $\mu(t)$ are arbitrary and do not depend on the parameter α, equation (A1) is equivalent to the maximized present value Hamiltonian evaluated in the first-best social optimum. Since $u(c^*, x^*)e^{-\theta t} = \overline{H} - \lambda\dot{k}^* - \mu\dot{x}^*$, and by applying the rules of partial integration, the optimal value function can be written as

$$V^*(0, \alpha) = \int_0^\infty [\overline{H}(t, \alpha) + \dot{\lambda}(t)k^*(t, \alpha) + \dot{\mu}(t)x^*(t, \alpha)]dt - \lambda(t)k^*(t, \alpha)|_0^\infty - \mu(t)x^*(t, \alpha)|_0^\infty. \tag{A2}$$

The cost–benefit rule we are looking for is derived by differentiating equation (A2) with respect to α and evaluating the resulting derivative in the first-best social optimum, where $\lambda(t) = \lambda^*(t, \alpha)$ and $\mu(t) = \mu^*(t, \alpha)$. Since $k(0)$ and $x(0)$ are fixed, and if we assume that the transversality conditions $\lim_{t\to\infty}\lambda(t) = 0$ and $\lim_{t\to\infty}\mu(t) = 0$ are fulfilled, we can write the cost–benefit rule as

$$\frac{\partial V^*(0, \alpha)}{\partial \alpha} = \int_0^\infty \left[\frac{\partial \overline{H}^*(t, \alpha)}{\partial \alpha} + \dot{\lambda}^*(t, \alpha)\frac{\partial k^*(t, \alpha)}{\partial \alpha} + \dot{\mu}^*(t, \alpha)\frac{\partial x^*(t, \alpha)}{\partial \alpha}\right]dt. \tag{A3}$$

By using the first-order conditions for c and g given by equations (9) and (10), the first term within the square bracket of equation (A3) can be written as (where the time indicator and the parameter α have been suppressed for notational convenience)

$$\frac{\partial \overline{H}^*}{\partial \alpha} = [u_x^* e^{-\theta t} - \mu^*\gamma]\frac{\partial x^*}{\partial \alpha} + \lambda^*\left[f_k^*\frac{\partial k^*}{\partial \alpha} + f_\alpha^* - I_\alpha(\alpha)\right] \tag{A4}$$

where $u_x^* = u_x(c^*, x^*)$, $f_k^* = f_k(k^*, g^*, \alpha)$ and $f_g^* = f_g(k^*, g^*, \alpha)$. Finally substitution of the equations of motion for $\lambda^*(t, \alpha)$ and $\mu^*(t, \alpha)$, given by equations (11) and (12), into equation (A3) gives equation (34).

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NOTES

1. Readers interested in more thorough literature reviews are referred to Aronsson, Johansson and Löfgren (1997), Weitzman (2003) and Aronsson, Löfgren and Backlund (2004).
2. Other early theoretical contributions to the study of social accounting are, for example, Hartwick (1990) and Mäler (1991), who analysed Weitzman's welfare measure in the context of economies where environmental and/or natural resources are important components of the economic system. See also Aronsson and Löfgren (1996), who consider welfare measurement in an economy with human capital externalities.
3. Welfare measurement problems in economies with heterogeneous consumers are addressed by, for example, Aronsson and Löfgren (1999b) and Aronsson (2008).
4. This is seen by observing that $\partial H/\partial c = 0$, $\partial H/\partial g = 0$, $d\lambda/dt = -\partial H/\partial k$ and $d\mu/dt = -\partial H/\partial x$ from the necessary conditions set out above, while $\partial H/\partial\lambda = dk/dt$ and $\partial H/\partial\mu = dx/dt$.
5. See Michel (1982).
6. If the technology is not stationary, then equation (14) may no longer apply. Löfgren (1992) and Aronsson and Löfgren (1993) consider a technology where the production function can be written as $f(k(t), g(t), t)$, where the direct effect of time is interpretable in terms of disembodied technological change. With this seemingly innocent modification, they show that the welfare measure in equation (14) changes to read

$$\theta V^*(t) = H^{c^*}(t) + \int_t^\infty \lambda^{c^*}(s) f_s(k^*(s), g^*(s), s) e^{-\theta(s-t)} ds$$

   where the second term on the right-hand side measures the present value of marginal technological change. In this case, it follows that (i) the current value Hamiltonian no longer constitutes an exact welfare measure, and (ii) information referring to time t no longer suffices to measure welfare at time t. See also Kemp and Long (1982).
7. The marginal value is measured as the marginal rate of substitution between pollution and private consumption at time t, that is, the marginal willingness to pay – in terms of lost private consumption – for a reduction in the stock of pollution at that particular time, *ceteris paribus*. For an early attempt to apply the willingness to pay technique in the context of social accounting, see Peskin and Peskin (1978).
8. Weitzman (2001) and Li and Löfgren (2002) explain the role of the consumer surplus in social accounting in greater detail. See also Aronsson, Löfgren and Backlund (2004).
9. In Weitzman's study, the extra term vanishes, as he uses Stratonovich integrals instead of Ito integrals.
10. For further study of cost–benefit analysis in dynamic general equilibrium models, see, for example, Aronsson, Johansson and Löfgren (1997), Dasgupta and Mäler (2000), Arrow, Dasgupta and Mäler (2003) and Li and Löfgren (2008).
11. Seierstad (1981), Léonard (1987), Caputo (1990) and LaFrance and Barney (1991) have all given important contributions by eliciting formal cost–benefit rules for parametric changes in optimal control models. See also Seierstad and Sydsaeter (1987).
12. By solving equation (3) for $x^0(t, \vartheta)$ defined above, we obtain

$$x^0(t, \vartheta) = x(0)e^{-\gamma t} + \int_0^t g^0(s, \vartheta) e^{-\gamma(t-s)} ds,$$

   which means that

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$$\frac{\partial x^0(t,\vartheta)}{\partial \vartheta} = \int_0^t \frac{\partial g^0(s,\vartheta)}{\partial \vartheta} e^{-\gamma(t-s)} ds < 0$$

since $\partial g^0(t,\vartheta)/\partial\vartheta < 0$ for all t.

13. Although Weitzman (1976) did not attempt to examine genuine saving, it is, nevertheless, implicit in his study, as we need Weitzman's welfare measure (or an analogue thereof in the imperfect market economy) to relate the change in welfare over a short time interval to the genuine saving. Standard references for genuine saving are Pearce and Atkinson (1993) and Hamilton (1994, 1996).
14. Both uninternalized externalities and disembodied technological change contribute to make the economic system non-autonomously time-dependent, which means that both of them give rise to forward-looking terms in the welfare and genuine saving measures, simply because time itself has a direct effect on welfare that does not vanish from the optimal choices made by consumers and firms.
15. In fact, Asheim (1994) and Pezzey (1993) show that even if the genuine saving is non-negative at a particular point in time, the instantaneous consumption (or instantaneous utility more generally) may actually be declining.

REFERENCES

Aronsson, T. (2008) Social Accounting and the Public Sector. *International Economics Review* **49**, 349–375.

Aronsson, T., Johansson, P.-O. and Löfgren, K.-G. (1997) *Welfare Measurement, Sustainability and Green National Accounting: A Growth Theoretical Approach*. Cheltenham: Edward Elgar Publishing Limited.

Aronsson, T. and Löfgren, K.-G. (1993) Welfare Consequences of Technological and Environmental Externalities in the Ramsey Growth Model. *Natural Resource Modeling* **7**, 1–14.

Aronsson, T. and Löfgren, K.-G. (1995) National Product Related Welfare Measures in the Presence of Technological Change, Externalities and Uncertainty. *Environmental and Resource Economics* **5**, 321–332.

Aronsson, T. and Löfgren, K.-G. (1996) Social Accounting and Welfare Measurement in a Growth Model with Human Capital. *Scandinavian Journal of Economics* **98**, 185–201.

Aronsson, T. and Löfgren, K.-G. (1999a) Pollution Tax Design and Green National Accounting. *European Economic Review* **43**, 1457–1474.

Aronsson, T. and Löfgren, K.-G. (1999b) Welfare Equivalent NNP under Distributional Objectives. *Economics Letters* **63**, 239–243.

Aronsson, T., Löfgren, K.-G. and Backlund, K. (2004) *Welfare Measurement in Imperfect Markets: A Growth Theoretical Approach*. Cheltenham: Edward Elgar Publishing Limited.

Arrow, K., Dasgupta, P. and Mäler, K.-G. (2003) Evaluating Projects and Assessing Sustainable Development in Imperfect Economies. *Environmental and Resource Economics* **26**, 647–685.

Asheim, G.B. (1994) Net National Product as an Indicator of Sustainability. *Scandinavian Journal of Economics* **96**, 257–265.

Asheim, G.B. (2007) Can NNP be Used for Welfare Comparisons? *Environment and Development Economics* 12, 11–31.

Brock, W.A. (1977) A Polluted Golden Age. In Smith, V.L. (ed.) *Economics of Natural and Environmental Resources*, New York: Gordon and Breach.

Caputo, M.R. (1990) How to do Comparative Dynamics on the Back of an Envelope in Optimal Control Theory. *Journal of Economic Dynamics and Control* **14**, 655–683.

Dasgupta, P. and Mäler, K.-G. (2000) Net National Product, Wealth and Social Well-being. *Environment and Development Economics* **5**, 69–93.

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Dixit, A., Hammond, P. and Hoel, M. (1980) On Hartwick's Rule for Regular Maximin Paths of Capital Accumulation. *Review of Economic Studies* **47**, 551–556.
Hamilton, K. (1994) Green Adjustments to GDP. *Resources Policy* **20**, 155–168.
Hamilton, K. (1996) Pollution and Pollution Abatement in the National Accounts. *Review of Income and Wealth* 42(1), 13–33.
Hartwick, J. (1977) Intergenerational Equity and the Investing of Rents from Exhaustible Resources. *American Economic Review* **66**, 972–974.
Hartwick, J. (1990) Natural Resources, National Accounting and Economic Depreciation. *Journal of Public Economics* **43**, 291–304.
Kemp, M.C. and Long, N.V. (1982) On the Evolution of Social Income in a Dynamic Economy: Variations on a Samuelsonian Theme. In Feiwell, G.R. (ed.) *Samuelson and Neoclassical Economics*, Boston, MA: Kluwer-Nijhoff.
LaFrance, J.T. and Barney, L.D. (1991), The Envelope Theorem in Dynamic Optimization. *Journal of Economic Dynamics Control* **15**, 355–385.
Léonard, D. (1987) Costate Variables Correctly Value Stocks at Each Instant. *Journal of Economic Dynamics and Control* **11**, 117–122.
Li, C.-Z. and Löfgren, K.-G. (2002) *On the Choice of Metrics in Dynamic Welfare Analysis: Utility versus Money Measures*. Umeå Economic Studies no. 590, Umeå: University of Umeå.
Li, C.-Z. and Löfgren, K.-G. (2008) Evaluating Projects in a Dynamic Economy: Some New Envelope Results. *German Economic Review* **9**, 1–16.
Löfgren, K.-G. (1992) Comments on C.R. Hulten, Accounting for the Wealth of Nations: The Net Gross Output Controversy and its Ramifications, *Scandinavian Journal of Economics*, 94 (suppl.), 25–28.
Mäler, K.-G. (1991) National Accounts and Environmental Resources. *Environmental and Resource Economics* **1**, 1–15.
Michel, P. (1982) On the Transversality Conditions in Infinite Horizon Optimal Control Problems. *Econometrica* **50**, 975–985.
Pearce, D.W. and Atkinson, G. (1993) Capital Theory and the Measurement of Sustainable Development: An Indicator of Weak Sustainability. *Ecological Economics* **8**, 103–108.
Peskin, H.M. and Peskin, J. (1978) The Valuation of Nonmarket Goods in Income Accounting. *The Review of Income and Wealth* **24**, 71–91.
Pezzey, J. (1993) *The Optimal Sustainable Depletion of Nonrenewable Resources*. London: University College, London.
Samuelson, P.A. (1961) The Evaluation of Social Income: Capital Formation and Wealth. In Lutz, F.A. and Hague, D. (eds) *The Theory of Capital*, New York: St. Martin's Press.
Seierstad, A. (1981) Derivatives and Subderivatives of the Optimal Value Function in Control Theory. Memorandum from Institute of Economics, University of Oslo.
Seierstad, A. and Sydsaeter, K. (1987) *Optimal Control Theory with Economic Applications*. Amsterdam: North-Holland.
United Nations, Commission of the European Communities, International Monetary Fund, Organisation for Economic Cooperation and Development and World Bank (2003), *Handbook for Integrated Environmental and Economic Accounting* New York: UN. Draft available at http://unstats.un.org/unsd/envaccounting/seea2003.pdf, accessed 16 May 2010.
Weitzman, M.L. (1976) On the Welfare Significance of National Product in a Dynamic Economy. *Quarterly Journal of Economics* **90**, 156–162.
Weitzman, M.L. (1998) *Comprehensive NDP and the Sustainability Equivalence Principle*. Cambridge, MA: Harvard University Press.
Weitzman, M.L. (2001) A Contribution to the Theory of Welfare Accounting. *Scandinavian Journal of Economics* **103**, 1–24.
Weitzman, M.L. (2003) *Income, Capital, and the Maximum Principle*. Cambridge, MA: Harvard University Press.
World Commission on Environment and Development (1987) *Our Common Future*. Oxford: Oxford University Press.

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