Introduction

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If a symbolic date were to be chosen for the birth of mathematical economics, our profession, in rare unanimous agreement, would select 1838, the year in which Augustin Cournot published his *Recherches sur les principes mathématiques de la théorie des richesses*.

Gerard Debreu (1984, p. 267)

Economics has secured the posterity of Augustin Cournot's works, but there was a large temporal gap between their application and their original publication. Cournot's role in applying mathematics to the social sciences is an exceptional, and perhaps unique, contribution. Clearly ahead of his time, the nineteenth-century French mathematician found almost no interlocutors. The engineer-economists, for whom he wrote, had no need for his abstract and general approach, and the theoreticians simply ignored him. It was not until Walras, Jevons, Marshall or Pareto that Cournot found attentive readers who were eager to further his work. It is without a doubt Irving Fisher who introduced Cournot to economists in his 1898 commentary on the English translation of *Recherches sur les principes mathématiques de la théorie des richesses*, exactly 60 years after its original publication in French. In the second half of the twentieth century, the triumph of game theory consecrated Cournot's market theories, which were nonetheless marked by the ambiguities inherent in such a time lag. Finally, in 2008, Cournot's name will appear for the first time on the list of authors on secondary school biology syllabi in France.

Cournot's works have had such a vast influence on the social sciences that it would be futile to try to separate that which he would have recognized as a continuation of his work from that which he would have rejected. In and of itself, the question is of little interest, given that Cournot remained remarkably detached from his writings. The question gains pertinence, however, when one considers the contribution that his works have made to founding a normative discourse, a discourse that Cournot did not participate in. The aim of the conference proceedings presented in this book, however, is neither to legitimate one particular approach to research born from his works, nor to prove his paternity to developments born of his intuitions. The aim is above all to pay homage to

Cournot's originality and modernity in the area of social mathematics, and more precisely in the field of economics.

The specificity of Augustin Cournot's work does not reside in his use of mathematics to describe the properties of social phenomena. During his life, Cournot only made irregular contributions to the 'theory of wealth', and the very nature of the subjects of the moral sciences, as they were called at the time, invites quantification. He was, however, the first to construct a mathematical model enabling those subjects to be treated analytically. In *Recherches sur les principes mathématiques de la théorie des richesses* (1838) (published in English as *Researches into the Mathematical Principles of the Theory of Wealth* in 1897), he set out to establish that 'the solution of the general questions which arise from the theory of wealth, depends essentially not on elementary algebra, but on that branch of analysis which comprises arbitrary functions' (p.4). Cournot undertook this task 'from a purely abstract standpoint, without reference to proposed applications' (p.5). While Cournot is certainly not the Galileo of economics, we must recognize that, just as the founder of classical physics broke with his predecessors, not by applying mathematical instruments to physical reality, but by upholding the mathematization of its phenomena as an inescapable methodological principle, Cournot's approach initiated the mathematical modelling of social phenomena.

At the same time, he indicated the fundamental orientation of its relation to mathematics. He was not concerned with producing new results in that discipline, as a scientist seeking to invent or discover new theorems would be, or with applying them to a particular object, as an engineer would be, so much as with considering, in a theoretical and reflexive manner, the applicability of mathematics to phenomena, in other words with exploring its conditions and limits. Cournot was profoundly convinced of the intelligibility of mathematics. This was not prompted by mysticism or naïve Pythagorism on Cournot's part, first because he never reduced mathematics to the quantitative dimension, and second because he was clearly aware of the diversity of the subjects and methods that makes mathematics so rich.

The mechanism of knowledge is essentially a process of ordering. This may be limited to a distribution of objects into distinct categories according to various principles of classification, placing the objects in their reciprocal exteriority. This represents a first level of organization, necessary but not sufficient for the intelligibility of these objects, for it is no more than descriptive. We take a further step forward when we seek to clarify the relations of dependence and subordination between ideas, and between phenomena, on the basis of their constituent properties. Not only is such an order constructed logically; it is also, according to Cournot, *rational*, in the sense that we can use it to explain things. Not that this rational order can mysteriously reveal to us the very essence of things (assuming that this term actually means anything), but by identifying the

multiplicity of relations by which objects are linked, it reveals the different ways and degrees to which they are interdependent. What Cournot called the ‘fundamental ideas of mathematics’ play a decisive role in this ordering: concepts such as number, distance, volume and so on, on which the mathematical sciences rely. In *Essai sur les fondements de nos connaissances et sur les caractères de la critique philosophique* (1851), he specified that it ‘is to be noted that many of these ideas, in spite of their high degree of generality and abstraction, are only particular forms and, as it were, concrete species of ideas still more abstract and general’ (1956 [1851], p. 233), such as combination, order, symmetry, inclusion, exclusion and so on.

Because of their high degree of generality, these ‘even more abstract’ ideas reach beyond the field of mathematics. They also form the basis of logic, and thus constitute the foundation of the formal sciences, which are characterized by their independence from any specific content or empirical determination. Cournot could not express himself in these exact terms, but this is the very conception that he proposed by invoking a ‘theory of order and form’. Moreover, it is precisely because of their independence, and therefore their formal character, that mathematical ideas are so fertile. These fundamental mathematical ideas designate formal, general relations between symbols, which can then be applied to the study of the relations between phenomena.

What is true for natural phenomena is equally valid for economic and social events. This is a first consequence of the above argument. The formal and symbolic character of mathematical ideas liberates them from any specific empirical content, so that they are capable of describing the diverse forms of relations between elements in any domain. Cournot actually applied this thesis, before formulating it explicitly, as early as 1838, in *Recherches*.

This does not mean, however, that the various mathematical sciences can be successfully applied indiscriminately to each domain of objects, or, to put it more clearly, to each of the different disciplines they study. We know that Cournot attached particular importance to two main branches of mathematics: infinitesimal calculus on the one hand, and probability calculus and statistics on the other. Two questions then arise. First, how did Cournot himself envisage the fertility of these instruments when applied to the investigation of social phenomena? Second, and more broadly, what has been the legacy of Cournot’s development of these instruments in the field of economics? More specifically, what remains, today, of the impetus given by Cournot to the mathematical modelling of the economy?

One may wonder whether fundamental mathematical ideas are pure intellectual constructions enabling a set of operations to be performed on the symbols that they both connect and designate, thanks to which we can then identify and classify the relations between phenomena, or whether these ideas are discovered by the mathematician, rather than invented, because they stem from the very

‘nature of things’ and are somehow ‘realized’ by phenomenal relations. Whatever Cournot’s position on this point, he believed that fundamental mathematical ideas formally designate general relations of order that *correspond* to the general relations that link phenomena to each other, and thus bring intelligibility. ‘The mind’, he wrote in 1872 in *Considérations sur la marche des idées et des événements dans les temps modernes*, ‘discovers mathematical truths through its own powers, and considers them as necessary truths: after which, in fact much later, the observer proves that the truths thus discovered do indeed explain and govern natural events’ (1973, p. 415–16).

Finally, the central motivation driving Cournot’s epistemological reflection is the desire to elucidate the significance and scope that can be attached to the applications of mathematics. He stressed this problem repeatedly: whereas mathematicians easily agree about the methods that should be used to obtain a result, they differ about the way that we should interpret its application to experience. Above all, Cournot devoted himself to forestalling the risk of confusion between an artificial process and a property of the object under study, or of the mistaken projection onto the object of characteristics inherent to the conceptual instrument used and the way it is implemented. Here, Cournot displayed an epistemological perceptiveness, the continuations and echoes of which we shall seek to discern in contemporary thought.

Our aim is not historiographical. We do not seek to reproduce Cournot’s thought as faithfully as possible for its own sake by comparing the diverse interpretations to which it has given rise, but instead to assess the relevance of his heritage, and therefore his topicality, on the epistemological, methodological and doctrinal levels. There is nothing self-evident about this heritage. When *Recherches* was published in 1838, it went almost completely unnoticed. Cournot’s later economic writings, *Principes de la théorie des richesses* (1863) and *Revue sommaire des doctrines économiques* (1877), stripped of the mathematical apparatus, did not meet with any warmer reception. Yet in the early 1940s, his contributions to economic theory were widely recognized and discussed. To what can we attribute this tardy recognition? What do we retain, today, from Cournot’s economic work? And, more broadly, how does it improve our conception of the relation between mathematics and experience?

These are the questions we shall be addressing in this work, in which economists, mathematicians, philosophers and statisticians have been invited to measure the legacy of Cournot’s work in the twenty-first century. The probabilistic approach lies at the heart of Cournot’s thought, not only because, as a mathematician, he wrote a treaty devoted to this branch of the discipline, *Exposition de la théorie des chances et des probabilités* (1843), but also because his epistemology is founded on the assertion that the relation of the theoretical hypotheses that constitute scientific knowledge to empirical reality can only be affirmed probabilistically, philosophical reflection then serving to evaluate, in

each case, the strength of that probability. Thus, it was as a probabilistic philosopher that Cournot investigated the relation of probability theory to experience and the status of the mathematical concept of probability.

To start with, Jean Magnan de Bornier recalls the essential contributions of Cournot's work to the discipline of economics that took shape in the twentieth century. He describes how attitudes towards them were modified and enhanced by subsequent developments in economics, paying particular attention to the theory of markets, Cournot's analysis of taxation and the key concepts of marginal cost and elasticity. He underlines the historical importance, for the constitution of mathematical economics, of Cournot's pioneering work on modelling, even if this latter was of limited extension, showing both its fertility and its limits. The destiny of *Recherches* is of particular interest to Jean Magnan de Bornier, first within the life and work of Cournot himself, who appeared to be curiously unconcerned about the future of a work of whose originality he was nevertheless well aware, and then within the later developments of the science of economics, mainly in the context of marginalist theory and then, more recently, in that of game theory, where Cournot's position has been redefined.

For his part, Thierry Martin shows that Cournot's representation is structured by a double distinction, first between mathematical and philosophical probabilities, and second between objective and subjective probabilities. Cournot's originality here was to make explicit this duality in the meaning of mathematical probability, not in order to reject it by favouring one interpretation to the detriment of the other, but to accept it as two forms of the probabilistic approach, depending on the way it is applied to things. The difficulty resides in determining how to attribute an objective meaning to mathematical probability when this latter cannot be reduced to a frequentist premise. This is addressed by two major theses of Cournot's epistemology: his objectivist representation of chance on the one hand, and the principle of physical impossibility, the forerunner of what contemporary probabilists call 'Cournot's principle', on the other. As this same epistemological concern governs Cournot's exposition of statistics, Martin goes on to re-examine the primacy that Cournot accorded to differential calculus in the analysis of economic equilibrium, refuting the idea that this bears witness to Cournot's resistance to the use of statistics in the field of economics.

As Bernard Walliser shows, if we can regard Cournot as the initiator of mathematical modelling in economics, it is in the sense that he used the mathematical instrument to consider economic phenomena. His epistemological reflection is not deployed in the form of a theory of models. Thus, to put Cournot's heritage to the test, Bernard Walliser defines six essential functions of modelling and uses them to form the basis of a general epistemology of models as they are constructed today. He then subjects their products to a critical examination with the aim of bringing to light their strengths and limits, and their respective capacities for explaining natural or social phenomena. His analysis highlights the

diversity in forms of modelling according to the theoretical principles by which they are governed and the empirical domains to which they refer, according to the formalisms they employ and the social roles they fulfil, and according to the operational and educational mechanisms they bring into play. Over and above these differences, however, and thanks to the operational power and formal simplicity of the mathematical instrument used, models provide access to results that a literary account could hardly, if at all, reach. In Walliser's words, they enable 'reasoning to be pursued through other means'. Nevertheless, every model needs to be completed by reflexive analysis, with the aim of interpreting the results obtained.

Glenn Shafer takes as his subject 'Cournot's principle', according to which an event of very low or zero probability will not occur. This principle, employed particularly by French and Russian probabilists during the first half of the twentieth century, makes it possible to give a clear empirical content to applications of probability calculus. Shafer shows that it was precisely the neglect of this principle that obscured the practical significance of probabilities, and in particular their application to the hypothesis of efficient markets. The principle can in fact be incorporated into game theory, where it signifies that no strategy exists by which the bettor can multiply his stake by an infinite factor without risking bankruptcy. He starts by describing the ways in which the principle was mobilized both before and after Cournot made it explicit, first in the form of the moral certainty so dear to eighteenth-century probabilists (notably Bernoulli and Buffon), and later by Chuprov, Markov and, in the French tradition, Borel, Lévy and Fréchet. This leads the author to recall the debates between probabilists in Europe and the USA over the significance of probabilities, and the role played by Cournot's principle in Kolmogorov's axiomatization of probability. After presenting the principles of probabilistic game theory, and explaining how it provided probability calculus with a richer foundation than the theory of measurement, in keeping with the analysis of Shafer and Vovk (2001), he shows how Cournot's principle can, in this context, be applied to market prices to obtain a weaker, and therefore more realistic, hypothesis of market efficiency than the classic hypothesis, according to which price variations are a function of new information.

Robert Aumann's text is the reproduction of his 2005 Prize Lecture, a version of which was presented at the Cournot Centre's conference. The lecture, which explains his contribution to economic theory, and particularly to non-cooperative game theory, pays direct homage to the works of Cournot. In discerning the equilibrium that is today associated with John Nash, Cournot laid the mathematical foundations for the analysis of non-cooperative games more than a century before the topic was systematically examined.

In his contribution, Robert Solow explores Augustin Cournot's macroeconomic intuitions and the degree to which his works heralded the theoretical

current of which the author was to be one of the founding fathers. Solow's re-reading of *Recherches* leads him to a mixed review: Cournot's great intuitions are sometimes marred by simplifications that modern economists see as errors. Solow, however, shows how much the latter owe to Cournot in their understanding of themes as diverse as variations in social income, the profits of international trade, duopolies, oligopolies and forms of competition.

Alain Desrosières presents two strong ideas from Cournot's probabilistic and statistical epistemology, and describes how fertile they proved to be in the twentieth century: first, his distinction between the objective and subjective meanings of probabilities, and second, his discussion of the interpretation of 'cuts', in other words, the distribution of a population of objects into distinct statistical categories. Both of these ideas raise the question of the way we interpret the results of mathematical analysis. We cannot neglect this interpretation, Cournot argued, without exposing ourselves to areas of ambiguity or obscurity; mathematical analysis makes it possible to denounce such areas in advance. Alain Desrosières puts particular emphasis on the construction of statistical nomenclatures, stressing that this presupposes active intervention on the part of the statistician to define a space of equivalence, thanks to which objects become comparable, beyond their particularities, and so lend themselves to categorization and statistical treatment. As we are in the domain of the social sciences, the operation of quantification, differentiated from that of measurement, does not encounter an object already constituted and offered up to the operation of measuring; it must construct its object, which does not pre-exist the analysis. By means of three examples – probabilistic risk assessment, the evaluation and interpretation of macroeconomic aggregates, and the evaluation of public management – Alain Desrosières examines the epistemological and sociological conditions of the translation through which the convention of equivalence, which makes quantification possible, is established. His analysis is bent to the task of showing how, during the second half of the twentieth century, the social uses of probability and statistics helped to provide a *practical* solution to the epistemological difficulties raised by the enterprise of quantifying social phenomena by defining equivalence conventions that enable us to 'compare the incomparable'.

REFERENCES

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