Introduction

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1. THE BOOK PLAN

Economic theory is rich in figures and diagrams. This book was conceived in the belief that we do not know as much as we should about the role of these in the development of economic theory.

The book has two aims. One is to provide an account of the role of each of the chosen figures and diagrams in economic analysis. Figures are part of the basic toolbox of the modern economist. We have selected figures that have been prominent in the history of economic analysis and that are, with a few exceptions, still found in contemporary textbooks and research. Together the topics portrayed cover a large part of mainstream economic theory and analysis.

The second aim is to provide an account of their histories. Thus, it is partly an exercise in the history of economic thought. Klein (1995) has commented on the general use of geometry in economic theory. There are existing histories of some of the diagrams; for example, Humphrey (1992, 1993) has written essays on the history of the box diagram, the 'trade theorist's sacred diagram' (the subject of Chapter 42) and the Marshallian cross diagram (see Chapter 1). Others have commented upon the use of geometry by some economists – chiefly Alfred Marshall (see especially his biographers, Keynes, 1972 and Groenewegen, 1995) but also Lerner (Giraud, 2008) and Samuelson (Giraud, forthcoming) – or in some particular period (for example, Cook, 2005) or area of economics (for example, de Marchi, 2003).

Our selection of topics is of course a little arbitrary. Most suggested themselves as they are commonly regarded as essential knowledge. After an initial selection, we contacted potential authors. A number of them suggested other topics. We then had 'an embarrassment of riches', as one of our correspondents had predicted, and could accept only some of the suggestions.

All areas in the discipline of economics are covered but preference was given to figures that are widely used. The topics are divided into three areas: those dealing with a market for a single commodity, that is, partial equilibrium analysis, those dealing with multiple markets, that is, general equilibrium analysis, and those from macroeconomics. The selection includes some that arose from empirical research – such as the Phillips curve, the Kuznets curve, the Lorenz curve and the logistic growth curve. It does not cover time series graphics or devices to represent data such as pie diagrams and bar charts.[1] Consequently we do not include the justly celebrated work of Playfield, *The Commercial and Political Atlas*, published in 1786.

Within these three areas, the topics have been put in groups that seem useful in terms of a commonality or overlap between the topics. Thus, in the first group of single market analyses, we distinguish between 'basic tools of demand and supply curve analysis', topics used in welfare economics, and those which arose in the study of particular markets such as the backward-bending supply curve in labour markets, the cobweb diagram in agricultural markets and the Markowitz diagram in asset markets.

In a similar way, general equilibrium diagrams are grouped into 'basic tools of general equilibrium analysis' and the extension of general equilibrium analysis to open economies, that is, the standard theory of international trade in goods. Open economy general equilibrium economics has been a particularly fertile area for the generation of figures.

Macroeconomic figures are grouped into two categories, tools of 'macroeconomic analysis and stabilisation' and 'growth, income distribution and other topics'. The former include the Keynesian income determination diagram, the IS-LM diagram and its extension to open economies, the Fleming-Mundell diagram, the aggregate demand-aggregate supply diagram, the Phillips curve and the UV or Beveridge curve and the demand for money. All of these are fundamental to the way macroeconomists approach questions relating to the time path of macroeconomies.

For each topic we asked an expert who has written about it in the past to prepare an entry. We sought a diversity of authors to try and bring out all aspects of graphical analysis. They include leading theorists, leading historians of economic thought and some younger economists. They did bring a diversity of approaches. Some are more formal than others. Some devote more space to tracing the origin of figures and controversies surrounding them. Some reproduce the original figure in their area and some do not. Some trace later developments of the original concept or curve and applications to areas that were not contemplated by the originators. This diversity enriches our appreciation of the development of graphical methods in economics and their contribution to economic theory.

2. MATHEMATICAL AND METHODOLOGICAL ISSUES

Mathematical Issues

The frequent use of geometry in economics is not surprising. Most of the figures and diagrams are figures in coordinate geometry. Figures are representations of one or more functional relationships in Euclidean 2 space. Functions of one variable, which lend themselves to portrayal by graphs in two-dimensional space, appear regularly in all areas of economic theory. A few of our entries are not figures in coordinate geometry terms. This applies to the diagrams relating to circular flows and to some of the diagrams in location theory and graph theory.

More than ten of the figures in the book are single curves – the Engel curve, Phillips curve and so on. Curves come in many shapes. Variously the curves hinge on some feature such as a minimum (the U-shaped average cost curve) or a maximum (the Laffer curve or the inverted U-shaped Kuznets curves), a point of inflection (the logistic curve), a change in sign of the slope (the backward-bending labour supply curve) or a kink (the kinked demand curve).

Many figures have two curves: a demand and a supply curve, two reaction curves and so on. The point of intersection of these curves provides the solution to the model. In some of these, the curves are representations of a system of several equations that can be reduced to two equations (as in the IS-LM curves). Some diagrams are a set of many curves, for example, an isoquant or indifference map.

Economists have shown great ingenuity in devising figures and diagrams. In addition to graphs or figures containing interactions between two or more curves, they have produced boxes and more complex figures with multiple quadrants (see Chapter 43, on the four-quadrant diagram showing general equilibrium in the two-sector Heckscher-Ohlin model). The Edgeworth box (and the variants used in examining the production allocation, the Stolper-Samuelson box and the box of the integrated world equilibrium diagram) keep track of relationships between 12 variables, as Humphrey (1996) noted. Curves have been manipulated in many ways. Shifts of curves illustrate comparative static propositions. There are figures showing dynamic relationships, for example, stability analysis and the cobweb diagram.

Some figures have been borrowed from mathematics or physics. An example of the former is the unit simplex and an example of the latter is the phase diagram used in the analysis of stability of equilibria (see Chapter 2 by Blaug on stability of equilibrium).

Conventions in economic theory figures

Generally economists have followed the standard conventions of coordinate geometry but there are instances when they have constructed their figures differently.

Almost invariably economists draw the horizontal and vertical axes to the right and upwards from the origin respectively, using only the non-negative orthant of Euclidean space. This is because the quantities of goods produced and consumed are necessarily positive and, with few exceptions, prices are defined as positive (that is, zero or strictly positive). Consequently, graphs with prices and quantities on the axes are in the first quadrant of Euclidean space. One exception to this general rule is diagrams relating to excess demand (or supply), as in offer curve analysis in international trade.

There is one peculiarity in the use of geometry by economists by comparison with pure mathematics and some other disciplines. Universally today economists draw demand and supply curves with the dependent variable, the quantity demanded or supplied, on the horizontal axis, whereas in mathematics it is conventionally on the vertical axis. Cournot (1838) drew demand and supply curves with the dependent variables on the vertical axis. Dupuit and Jenkin after him used this convention too. However, the German economists Rau and Mangoldt and later the English economist Alfred Marshall, whose *Principles of Economics* was published in 1890, drew them the other way. (For the history of these curves, see Chapter 1 in this volume). Although it is not widely remembered today, Marshall (1920, pp. 83 n.1 and 286 n.2) interprets his demand and supply curve with price as the dependent variable. His terms 'demand price' and 'supply price' indicate the prices at which consumers will demand and producers will supply the quantities specified.[2] However, in his discussion of the elasticity of demand and at other times he switches to price as the independent variable. Later generations of English-speaking economists have followed Marshall's axis convention but treated price as the independent variable. There is another example in economics of this odd practice. In his presentation of the Laffer curve, in which tax revenue is plotted as a function of the tax rate, Laffer puts the dependent variable, tax revenue, on the horizontal axis.

Many economists use graphs of functions in a rather casual way in that they draw curves over an unspecified domain of the independent variable. In the case of a demand or supply curve, for example, the curves often do not extend to zero price and they stop at some arbitrary point. We do not then know the quantity demanded or supplied if the price is zero or if it goes to infinity, or even if it can go to infinity. Marshall, for example, in the diagram which puts together the demand and supply curves (Marshall, 1920, Figure 19) does not extend his demand and supply curves to a zero price. At the other end of the curve, we normally take it that the price is unbounded upwards. However, there are instances where the independent variable is bounded upwards.

It is notable that figures and diagrams in economic theory make only occasional use of geometric theorems and propositions relating to shapes such as triangles or rectangles and hyperboles. They are chiefly concerned with laying out relationships among variables.

Methodological Issues

Figures and diagrams have been used in economic theory in several ways. They have been used as a device to discover economic results; theorems or properties of models; or comparative static propositions and dynamic propositions. They have been used to prove some results. And they have been used as an expository device.

It is not easy to uncover the role of figures and diagrams in the discovery of results. Discovery could have been made through a figure or diagram, or algebra or logic or sheer intuition. No doubt figures have on occasion been the way in which an economist or economists first perceived some result. Two examples where figures seem to have provided the vehicle of discovery are the Engel curve and the Lorenz curve. Both are examples of curves which were fitted to data. Where the curve or curves are functions derived from economic models, however, the discoverer(s) rarely tell us how the result was first conceived or understood. One clear example of discovery by the use of geometry is Scitovsky curves, for there is no mathematics other than geometry in his paper. Another example where the diagram seems to have been the vehicle of discovery is Marshallian stability analysis. In some other cases, algebra is evidently the vehicle of discovery. Two examples are Slutsky derivation of the income and substitution effect[3] and Nash equilibrium. In these cases, the diagram came after the discovery of the result.

In his early work on international trade, written in 1879, Marshall (1930) used what he described as the 'method of diagrams':

> It happens with a few unimportant exceptions all the results which have been obtained by the application of mathematical methods to pure economic theory can be obtained independently by the method of diagrams. Diagrams represent simultaneously to the eye the chief forces which are at work, laid out, as it were, in a map; and thereby suggest results to which attention has not been directed by the use of methods of mathematical analysis. The method of diagrams can be freely used by every one who is capable of exact reasoning, even though he has no knowledge of mathematics.

Thus Marshall advocated the use of diagrams as a method of investigation or exploration.

The use of figures and diagrams as a vehicle of proof has a chequered history. Mathematicians have always warned of the danger of proving propositions by geometry. Although an advocate and frequent user of figures, Marshall (1920, p.129 n.2) warns of this danger: 'It is to be remembered that graphical illustrations are not proofs. They are merely pictures corresponding very roughly to the main conditions of certain problems.' The reason is that one cannot draw a curve without its having a particular shape which gives it particular properties. In proving a proposition, one needs to make sure there is nothing about the shape of the curve or curves, the properties, which is critical to the proposition. There is another way of expressing this problem. Any curve can be represented by a function. But this function has a particular form. Hence, one needs to make sure that the proposition holds for all acceptable functions, not merely the one which represents the arbitrary curve drawn.

One example will suffice, the Law of Diminishing Returns. This can be stated in terms of either marginal or average returns, and the returns can diminish everywhere or eventually. The requirements for it to hold in any of these forms are stringent and there are many false proofs of versions of the Law supposedly holding under certain conditions. There was a long string of articles in the *American Economic Review* from 1963 to 1970 devoted to proofs of the Law. One statement of the Law, the textbook by Stonier and Hague (1957, p.229) – which was widely used in the UK and elsewhere at that time – contended that the Law of (everywhere) Diminishing (marginal) Returns holds if the production function is linearly homogeneous. The proof is geometric. But it is false. They had drawn their isoquants as strictly concave from above (that is, the production function is strictly quasiconcave) and with positive marginal products, without realising that linear homogeneity plus these assumptions suffice but linear homogeneity alone does not. (For an extensive discussion of alternative necessary and sufficient conditions, see Eichhorn, 1978, Chapters 4 and 10 and references therein).

This salutary example raises the rather deep issue of exactly what has been proven by graphical analysis. There is no simple answer since what is proven by geometry varies from author to author.

Even when geometry is used as the only form of mathematics, the proof of any theorem or proposition or assertion may use a combination of verbal logic and geometry. Historians of mathematical thought have examined the logical processes of Ancient Greek mathematics, which is the origin of geometry (see Knorr, 1987 and Netz and Noel, 2007, Chapter 4). The Ancient Greek mathematicians did not use any equations. They used a combination of verbal logic and geometry and they used their geometry in a subtle way. Their diagrams are schematic, representing the qualitative features of an object, and form part of a proof. Twentieth century economists like Lerner and Scitovsky used geometry in much the same way.

For other economists, geometry is a supplement to a proof using algebra or calculus. Cournot uses geometry in this way. The analysis in his text uses algebra and calculus only.

The use of geometry as a device for discovery and proof has declined in recent decades. This is due mainly to changes in the nature of economic models. They have become more complex in terms of the number of variables and relations. Growing complexity has clearly played a role in all three areas of economic theory examined here. In macroeconomic theory, for example, the growing complexity of analysis of aggregate demand in the form of more explanatory variables of consumption and investment behaviour and the introduction of aggregate supply forced macroeconomists to expound their models in terms of systems of equations rather than figures.

One aspect of complexity is especially important in this context, namely, the dimensions of a model. In general equilibrium theory, one can state these dimensions in terms of the number of goods and factors and sometimes of agents (as in models of a continuum of agents) and, in models of international trade, of countries. The geometry of Edgeworth and Pareto was appropriate to a version of the model in which there were only two consumers consuming only two goods, though Edgeworth used this geometry cleverly to explore the situation with more than two consumers (see Creedy's entry, 'The role of numbers in competition', Chapter 32). The same is true of the geometry of the staple Heckscher-Ohlin model with only two goods, two factors and two countries as pursued by Lerner and Stolper-Samuelson and others. However, from about 1950 to 1980, the thrust of developments in the theory of international trade was to extend the dimensions of the model from 2x2x2 (two goods, two factors and two countries), or 'twoness' as it was called, to any number of goods and factors and countries. This required the development of new tools of duality theory. Modern textbooks of international trade theory use geometry strictly as a low-dimensional illustration, not as a device for discovery or proof. Similarly, from the late 19th century, there was a drive to develop utility maximisation models of household demand with $n>2$ goods. Two-dimensional geometry could handle only two goods and here it again became increasingly a low-dimensional illustration of the more general theory, as in, for example, the discussion of income and substitution effects in Hicks and Allen 1934.[4]

After Cournot's work began to be read more widely there was a debate about the relative merits of geometry and algebra. This debate resurfaced after the publication of Samuelson's *Foundations* in 1947, which was another major advance in the application of mathematics to economic theory. Some reviewers accepted geometry but questioned the use of algebra. In reply, Samuelson (1952, p. 59) insisted 'Geometry is a branch of mathematics. . .'.

The comparison of algebra and geometry as alternative mathematical approaches can be clarified best by considering closely the relationships between a graph (or curve) and the equation of a function. The equation of a function that represents a graph is unique but the graph that represents a function is not, unless the function has a specific form with no parameters.[5] A non-specific function $y = f(x)$ may have some minimal restrictions imposed on it. If it is a demand function, for example, it may be restricted to being continuous and downward sloping. One can draw an infinity of graphs with these restrictions. It is not necessary that a graph is less precise than a function. For every graph and its representing function, one can be equally precise about the properties of both. However, it is usual to indicate the domain of the function and restrictions which apply, whereas, as noted above, economists using graphs are often vague about both. Thus algebra has advantages of being more general and more precise.[6]

On the other hand, in his review of Samuelson's *Foundations*, Boulding (1948) argued that geometry was more suited to dealing with functions 'in the large'. This can be a real advantage when curves have complex shapes or there are several curves. It may be the reason for the popularity of some graphs as expository devices (see below).

It is in the third role, as an expository device, that diagrams or figures have excelled and continue to excel. Textbooks in microeconomic theory, macroeconomic theory or other areas of economic theory all continue to use figures freely. They are used as illustrations of models. But they are much more than illustrations. In many areas of economic theory, the way in which economists understand economic concepts and propositions is through figures and diagrams. What teacher of economic theory has not seen the dawn of understanding come over students when, failing to understand an exposition of some complex model in algebra or calculus, they are presented with a simple illustration? This is the power of figures, an expression in economics of the old dictum that 'one picture is worth a thousand words' (or should we say a dozen equations?)

The big advantage of a diagram, from the point of view of information conveyed, is that it brings together and juxtaposes information contained in two or more curves that cannot be easily comprehended in equations or words.[7] One can comprehend relationships among a number of variables (as in the box diagrams) or the effects of shifting curves or multiple equilibria more readily than in the corresponding algebra. This advantage has been increased by modern technologies. Textbooks today use multi-coloured diagrams to great effect and the delivery of diagrams in class-room from computer-based programs allow overlays and other graphical techniques that aid the exposition of complex ideas.

3. HISTORICAL ISSUES

Confusion and Controversies

Some of the historical accounts show that the origin of the figures involved is unclear. One surprising example is the isoquant. Despite the fundamental importance of isoquants to production theory and the thousands of economists who have used them, it is not certain who first drew an isoquant.

In some cases, the diagram which is used by contemporary economists is not the diagram first used in the development of the analysis. It emerged from earlier diagrams as a result of debate and clarification. Four examples in this book are the backward-bending labour supply curve (see King's entry, Chapter 20), Edgeworth's consumption box (see Creedy's entry, Chapter 31), the elasticity of substitution (see Dixon's entry, Chapter 4) and the four-quadrant diagram in international economics (see Panagariya's entry, Chapter 43). Similarly, in some cases, the names have changed over time.

In other cases, there are controversies about the nature or meaning of diagrams. In our selection, there is a much greater prevalence of controversy reported for our selection of macroeconomic figures compared to those of microeconomics or general equilibrium. In most of the entries in the Macroeconomics section, the authors document controversies relating to who did what and what they meant. By comparison, the disputes or controversies that have arisen in microeconomics and general equilibrium are less common and, with a few exceptions, less heated. One of the exceptions is the vigorous controversy surrounding the idea of a homogeneous function as a representation of a constant returns to scale technology, which was quite radical when first suggested in the late 19th century[8] (see Stigler, 1941, chapter 12).

What is it about the early diagrams in macroeconomics that made their interpretation so debatable and the debate so heated? One aspect is the complexity of the models in terms of the number of variables. There are alternative ways of representing these relationships graphically (as in the controversies surrounding the IS-LM and the IS-LM-BP curves). A related factor may be the mixture of graphical, verbal and algebraic analysis that characterises these models. Yet, in microeconomics this is often true too but it did not generally give rise to prolonged debate. There are secondary controversies concerning the interpretation of variables (for example, desired versus realised outputs and expenditures) and the adjustment from one equilibrium to another. The pervasive factor is the absence of a clear microeconomic foundation for the behaviour of consumers and investors. For some of the functions, there are multiple theories, for example, those relating to the determination of the interest rate and the many specifications of the consumption function. The description of relationships was often imprecise. This gave rise to arguments about the sign and magnitude of slopes of curves and their linearity or nonlinearity. On this score, see Richard Lipsey on aggregate demand aggregate supply analysis (Chapter 49), Warren Young on the IS-LM curve (Chapter 47) and Warren Young and Russell Boyer on the Fleming-Mundell extension (Chapter 48).

The First Figures and Diagrams in Economics

The early writings in economics were verbal, with the occasional use of numerical examples. Adam Smith's *Wealth of Nations*, published in 1776, does not contain a single figure. Nor do any of the editions of David Ricardo's *Principles of Political Economy and Taxation* published in 1817, 1819 and 1821, nor those of the editions of John Stuart Mill's *Principles of Political Economy* published between 1848 and 1871.

The first figure or diagram to appear in economics is the zigzag diagram of flows of income and expenditure in a rural economy produced by Quesnay in 1758. This is perhaps the most famous figure or diagram in economics, but there is still disagreement about its interpretation (see Chapter 29, on circular flow diagrams, by Backhouse and Giraud).

It is not certain when the first graph or figure appeared in economics. Theocharis (1983) traces a few examples of graphs in the 18th and early part of the 19th centuries, a time when economics was not yet a separate discipline. Daniel Bernouilli, writing in Latin in 1738, graphed utility as a logarithmic function of wealth (Bernouilli, 1738, reproduced in Baumol and Goldfeld, 1968). His now celebrated paper was finally published in English in *Econometrica* in 1954. His concave function is that of a risk-adverse individual in modern terms.

The French physiocrat, Pierre du Pont de Nemours, drew a diagram of 'the level of natural prices as they would have been in the absence of excise and as they would be after the suppression'. His curve follows a geometric progression. In fact, his curve has the same form as the total product curve when the Law of Diminishing Returns is a decreasing geometric series.

This simple approximation was used by von Thünen and many other agricultural economists and scientists. The continuous function which passes through these points is the Mitscherlich-Spillman function (see Lloyd, 1969).

The Italian Guiglielmo Silio, following the analysis of smuggling by his fellow Milanese Cesare Beccaria, in 1792 plotted a graph of the quantity of goods smuggled as a function of the total duty paid. This curve is a hyperbola though the axes are not displayed in the conventional vertical and horizontal locations (see Theocharis, 1983, p. 24).

The Frenchman Comte Germain Garnier drew a curious diagram in 1796, using a 'pyramid' rather than Cartesian axes. Garnier's pyramid is in reality an isosceles triangle in which the vertical distance is the price of a good. The horizontal width at a point of the Garnier pyramid represents either the number of people willing to buy the good or the share of wealth which each individual is willing to use for the purchase of the good. The first interpretation implies a very particular demand curve in which the number of buyers is a declining function of price. Translated into coordinate geometry, it is in fact a straight line.[9]

In 1830, the Englishman T. Perronet Thompson, when discussing a desirable scale of income taxation, plotted the tax payment as a proportion of income as a continuous function of income which approaches a maximum proportion asymptotically. The equation of the curve is $y=(x-a)/x$ (Theocharis, 1983, p. 123). In his figure Thompson shifts the origin to $(a,0)$ so that his curve passes through the origin.

These five are isolated cases in that each deals with a different problem that arose in public discussion of the time. The graphs are used as illustrations and all are graphs of simple specific functions that were known at the time. Only Bernouilli's curve follows modern conventions with respect to axes and the origin. Their occurrence in economic literature broadly parallels the increasing use of coordinate geometry in the physical sciences in the 18th and 19th centuries after the great work of Descartes, *Geometria*, published in 1637 (see Klein, 1995).

The introduction of graphical methods of analysing economic problems is due to the Frenchman Antoine-Augustin Cournot in *Recherches sur les Principes Mathématiques de la Theorie des Richesse (Researches into the Mathematical Principles of the Theory of Wealth)* published in 1838. He made enormous advances. Cournot used functions with no specific form.[10] He was the first economist to do so extensively.[11] He did this both in his treatment of demand and supply and in his analyses of monopoly and duopoly. He wrote the demand function as $D=\mathrm{f}(p)$, argued that it has the restrictions of being continuous and downward sloping and graphed it. He had an unspecified total revenue function, $pF(p)$. He also used the inverse demand function, in his analysis of duopoly and competition, which he wrote as p=f(D). On the supply side, he had a total cost and marginal cost function and the supply function, $\Omega(p)$, which he argued is upward sloping and which he graphed. Humphrey (1992, p. 4) states that 'Cournot . . . was the first to draw market demand and supply curves'. In his analysis of production, he introduced unspecified demand and net profit functions.[12] In his classic analysis of duopoly, he introduced unspecified reaction functions. These advances are colossal, but he was after all a mathematician who wrote a treatise on functions and infinitesimal calculus. There are a total of 10 diagrams in his book, all of which appear as a loose leaf section at the end. They show the demand curve, equilibrium in the case of duopoly, the 'Marshallian' scissors diagram and the incidence of a commodity tax.

We can also state that Cournot was the first economist to produce a figure in which curves intersect to determine a solution. He brought the downward-sloping demand and upward-sloping supply functions together in a single diagram for the first time to yield the market clearing price. Unfortunately, Cournot's work received little attention for several decades.

After 1850, the use of figures in economic theory spread steadily as all forms of mathematical analysis became more common in economic works. Figures play an important part in the work of English economists such as Jevons (who was one of the first English economists to discover the works of Cournot but only after he had done his major work) and Edgeworth. Jenkin used demand and supply curves in the way they are used today to examine market equilibria.

Marshall was a strong advocate of the use of diagrams. He cited the presentations of demand and supply curves by Cournot and Jenkin. He introduced possibilities of multiple and of unstable equilibria in his figures. In Marshall's *Principles*, algebra is relegated to the Mathematical Appendix and diagrams are the main form of mathematics, though all of these too are relegated to footnotes. This relegation reflects Marshall's strong views on the limitation of mathematics in economic theory (see Groenewegen, 1995, pp. 412–3). In the eighth edition, published in 1920, there are 41 numbered figures plus four more in the Mathematical Appendix.

In his biographical essay on Marshall, Keynes (1972, pp. 185–86) credits him with being the 'founder of modern diagrammatical economics':

> Marshall's mathematical and diagrammatical exercises in Economic Theory were of such a character in their grasp, comprehensiveness, and scientific accuracy, and went far beyond the 'bright ideas' of his predecessors, that we may justly claim him as the founder of modern diagrammatical economics.

As noted above, this credit should go to Cournot. He matched Marshall in every one of these respects and, moreover, he had no precedents at all to follow.

Marshall's *Principles* certainly did have a profound influence on the development of economic theory in the English-speaking world. One aspect of this influence is that later economists took many of the graphs and figures used thereafter in economic theory from his work, including downward-sloping demand curves and upward-sloping supply curves, the elasticity of demand, the marginal product function, the central 'Marshallian' cross diagram, derived demand, the incidence of a tax or bounty, the diagram of rent determination and the diagram showing consumers' and producers' surplus. In *The Pure Theory of International Trade*, published in 1879, he introduced offer curves, though he called them 'international trade' curves.

The Chronology of our Figures and Diagrams

Appendix 0.1 contains a chronology of the figures and diagrams covered in this book. They are presented in the same order as in the book. Further detail of discovery can be found in the individual entries.

In some cases, one or more authors presented a new concept in algebra and a later author or authors presented it in terms of geometry. An initial algebraic statement in such cases is indicated by square brackets. In other cases, the listing is debatable as there is no clear single discoverer or discoverers. For some, there are antecedents which have not been listed because the form of the antecedent diagrams differed from that of the diagram used today; for example, a number of writers had presented the essential ideal of the Laffer Curve for various kinds of taxes.

Despite the complexity and uncertainty with respect to the origin of some curves and diagrams, the general pattern of the evolution of these figures is clear. The first figures arose in microeconomic theory, apart from Quesnay's circular flow diagram in 1758. The very first diagram in microeconomics listed in the table is that of von Thünen's location theory in 1826. Cournot's figures analysing market behaviour and the logistic growth curve of Verhulst both appeared in 1838. Thereafter there was a steady stream of new figures and diagrams in microeconomic theory. Figures illustrating aspects of general equilibrium in closed or open economy began to appear from the 1880s, after Walras had developed the theory. In macroeconomic theory, surprisingly, diagrams did not appear until after the publication of Keynes's *General Theory*, apart from Marshall's curve relating to the demand for money.

The modal decade in terms of the number of figures that first appeared is the 1930s by a large margin (most of these are by British or British-trained authors, reflecting the influence of Marshall). Figures continued to appear regularly for the first time in the 1940s and 1950s and with decreasing frequency in the subsequent decades. The last diagram we have listed is that in graph theory in 1996 by Economides.

Another result from this chronology is that two kinds of multiple discoveries can be observed. These are multiples discoveries of a curve or diagram, and multiple discoveries of different curves or diagrams by one economist.

Multiple Discoveries and Eponomy

Eponomy

Eponomy is 'the practice of affixing the name of the scientist to all or part of what he has found, as with the Copernican system, Hookes' law, Planck's constant, or Halley's comet' (Merton, 1973, p. 299). This practice is common to all sciences. The naming is done by someone after the event, often a long time after, as a form of public recognition of the contribution.

Segara and Braun (2004) have produced a dictionary of eponyms in economics, that is laws, theorems, functions, equations and other entities named after their discoverer or discoverers, or at least a person or persons credited with their discovery. There are over 300 in their list. This includes the great majority of those named in our collection (starting with the Marshallian cross diagram, the Engel curve and so on) though there are some omissions – the Lerner diagram, Beveridge curve and Fisher's diagram – all of which are surprising.

The eminent sociologist of science, Robert Merton (1973, p. 356), has examined the application of eponomy to scientific discoveries. He advanced the hypothesis that 'all scientific discoveries are in principle multiples, including those that on the surface appear to be singletons', meaning that they were 'in the air' and not the product of unique genius.

In the literature on the history of economic thought, multiple discoveries have been noted many times. In discussing one such example, the origin of the marginal revenue curve, Andrew Skinner cites Joan Robinson's vivid metaphor: 'There are many occasions when several explorers are surprised, on meeting each other at the pole' (Joan Robinson, 1933, pp. vi–vii). George Stigler (1980), the eminent historian of economic thought, gives several examples of multiple discoveries in economics. These include three in our set: the Slutsky equation, the kinked oligopoly demand curve and monopolistic and imperfect competition (though in this instance, there are important differences between the models of Edward Chamberlin and

Joan Robinson). Several more of our authors found that more than one person independently discovered or invented the graph or diagram they report, for example, the utility possibility curves.

Stephen Stigler, statistician and son of George Stigler, modified the Merton thesis. He advanced Stigler's Law of Eponomy: 'No scientific discovery is named after its original discoverer' (Stigler, 1980, p. 147). The person named is someone who made the discovery after an earlier discoverer, sometimes when the earlier work was known to the person named. In a few cases the named discoverer did not even make the discovery. A well-known example of this in economics is Giffen's paradox and a lesser known example is the Beveridge curve. A more famous example is Pythagoras's Theorem.[13]

Stigler's Law, which he asserts holds with the same generality as other laws in sciences, certainly holds for at least three of the curves or diagrams named in this collection. It holds for the Marshallian cross diagram. In his entry on this diagram in Chapter 1, Humphrey examines in detail five other economists who used this diagram before Marshall. Marshall himself acknowledges the work of four of these, Cournot, Dupuit, Mangoldt and Jenkin. It holds for the Edgeworth box too; the four-sided box first appeared in Pareto's 1960 *Manual*. It also holds for the Stolper-Samuelson box. This box was presented by Lerner in 1933. However, there is no suggestion that Stolper and Samuelson knew of the earlier discovery: Lerner's piece was presented as a seminar paper at the London School of Economics in 1933 but was not published until 1952, after the article by Stolper and Samuelson. Stigler's Law also holds for the Laffer curve. There are several antecedents of this curve. In this instance, as noted by Roger Middleton, Laffer (2004) admitted these antecedents and claimed only to be a populariser.

On the other hand, there are more instances in this collection of eponymous namings which, contrary to Stigler's Law, are to the original discoverer. The first of these is the Engel curve. The next, in time sequence, are Edgeworth's indifference curve, the Lerner diagram, and Nash equilibrium. No one, to our knowledge, has provided evidence of a Stiglerian predecessor in these instances.

In some cases too, the multiple diagrams are slightly different but closely related diagrams; examples of this are Marshallian and Walrasian stability analyses, the two variants of monopolistic competition, the various classifications of technical change and various rent-seeking diagrams. In the case of circular flow diagrams, these have arisen in quite different contexts – general equilibrium and macroeconomic models of an economy – and therefore look different, but the essential idea of diagrams showing two-way flows is the same.

Economists who made multiple discoveries

The second kind of multiple discoveries occurs when one economist makes multiple discoveries. In this volume, several economists are credited with discoveries of multiple figures, in some cases along with but independently of other discoverers. For example, Cournot is credited, before several other independent discoverers, with demand and supply curves, the 'Marshallian' cross diagram and 'Cournot' duopoly. Marshall is credited with the stability diagram, the phase diagram, the monopoly diagram and the demand for money diagram. Edgeworth is credited with the indifference curve, the Edgeworth box and the first presentation of u-shaped average cost curves (see Chapter 8). Hicks is credited with discovering the elasticity of substitution, the income and substitution effect diagram (jointly with Allen), the IS-LM diagram and the 'Hicksian' form of technological change. Joan Robinson is credited with the elasticity of substitution, the diagrams of discriminating monopoly and imperfect competition, and the backward-bending labour supply curve. Samuelson is credited jointly with Stolper with the production box, the utility possibility curve, the factor price frontier and the Keynesian cross diagram. Solow is credited with the Solow classification of technical change, the Solow model and one of the diagrams showing the non-neutrality of money (jointly with McDonald). Leontief and Kuznets are credited with two discoveries, though the Kuznets environmental curve is named after him because of its similarity to the Kuznets curve describing the relationship between the level of per capita income and income inequality. These multiple discoveries are one aspect of the creativity of great economists.

The star performer in our collection, in this respect, is Abba Lerner. He is credited with the Lerner diagram and the box construction for the analysis of the production side of an economy, and is recognised (in Chapter 3 by Lloyd and Blaug) as one of the first economists to draw an isoquant. He was the originator of the Lerner degree of monopoly in Lerner (1934) and its diagram. Lerner is now recognised as one of the most prolific and productive users of geometry in the history of economic analysis (see Giraud, 2008).

4. FINAL REMARKS

This review of the mathematical, methodological and historical aspects of the use of figures and diagrams shows that they have played a central role in the development of economic theory. First, they have been a major vehicle of discovery of economic concepts and propositions, although the process of discovery in some cases was confused. Second, they have excelled as an expository device. There are some people whose visual sense is stronger than their sense of prose or their sense of numbers. For others, complex relationships can be comprehended more clearly in terms of figures. If you are one of these, this is a book for you. It teaches economics by looking at curves and shapes.

NOTES

1. Maas and Morgan (2002) give an interesting history of graphs of time series in 19th century Britain. The use of graphs was contentious in the beginning of the history of statistics and econometrics.
2. In the Mathematical Appendix to the *Principles*, Marshall writes the (market) demand and the supply function in the inverse price form. Gordon (1982) traces Marshall's ambivalence with respect to the choice of axes and attributes his preference for the inverse form to his treatment of consumers' surplus. Here price is the maximum price consumers are willing to pay for a given quantity.
3. Hicks and Allen seem to have come to the geometry first. In presenting the diagram, they say 'Here we may conveniently begin with a geometrical treatment, concentrating on the case where income is spent on two goods only – the case most amenable to the geometrical method' (Hicks and Allen, 1934, p. 65). They derive a different form of the Slutsky equation. The terms 'income effect' and 'substitution effect' are not used until Hicks's *Value and Capital,* published in 1938, where the treatment is again geometrical.
4. Earlier Edgeworth had made the same point: 'The mathematical version of the theory consists either of Geometry or Algebra. Geometry is directly applicable to the simplest possible cases. If more than two commodities are considered, solid geometry must be called in. The dimensions of space are not adequate to represent the case of more than three variables' (Edgeworth, 1894, p. 424).
5. For example, $y = \log x$. For a straight line, $y = a+bx$, there is a multiple infinity of lines as the parameters a and b may vary from $-\infty$ to $+\infty$. Once the values of a and b are specified, there is a unique graph.
6. Boulding (1948) also argued that geometry was a more convenient way of dealing with discontinuities. This is arguable and, in any case, discontinuities do not figure frequently in economic theory (one exception is the kinked demand curve).
7. Larkin and Simon (1987) compare the computational efficiency of diagrams and equivalent words arranged in sentences, using information theory. They argue that diagrams can be superior to words because they group information together, use the location of information and support perceptual inferences. They do not compare diagrams and equivalent equations.
8. Edgeworth made the celebrated jibe: 'Justice is a perfect cube, said the ancient sage: and rational conduct is a homogeneous function, adds the modern savant' (quoted in Stigler, 1941, p. 342). This comment was made by one of the most skilful geometricians in the history of economic analysis and the best mathematical economist of his time.
9. Let x denote the number of buyers, p the price, b the base of the triangle and a its height. Then $x = b-(b/a)p$, with $p\geq 0$ and $x \geq 0$.
10. In the Preface, he wrote, 'I propose to show that the solution of the general questions which arise from the theory of wealth, depends essentially not on elementary algebra, but on that branch of analysis which comprises arbitrary functions, which are merely restricted to satisfying certain conditions'.
11. Baumol and Goldfield (1968, p.14) comment that 'As far as we know, it was not until (almost exactly) 100 years after the publication of Bernouilli's essay that functions of

unspecified form were first introduced into economics (by Cournot)'. Certainly, the five curves in the text above that were presented before Cournot are all graphs of specific functions. However, this distinction should go to the German Georg von Buquoy. In 1815 he used non-specific revenue and cost functions and used the equality of marginal revenue and marginal cost to determine the optimal level of ploughing a field. (Theocharis, 1983, p. 111).

12. Baumol and Goldfeld (1968, pp. 162–63) praise a number of features of his careful mathematical analysis of monopoly.
13. On the basis of recent scholarship, Stigler (1980, p. 148) claims that 'the Pythagorean Theorem, was known before Pythagoras, was first proved after Pythagoras, and in fact Pythagoras himself may have been unaware of the geometrical significance of the theorem!'

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APPENDIX 0.1 A CHRONOLOGY OF CURVES AND DIAGRAMS

Curve/diagram	Discoverer(s)	Date
Microeconomics		
'Marshallian' cross	Cournot	1838
	Rau	1841
	Dupuit	1844
	Mangoldt	1863
	Jenkin	1870
Stability of equilibrium	Walras	1874
	Marshall	1879
Indifference curves	Edgeworth	1881
Isoquants	Bowley	1924
Elasticity of substitution	Hicks	1932
	Robinson	1933
Income and substitution effects	[Slutsky]	1915
	Hicks and Allen	1934
Engel curves	Engel	1857
Homogeneous functions	[Wicksteed]	1894
	[Flux]	1894
Homothetic functions	Shephard	1970
Long-run and short-run curves	Edgeworth	1913
Product exhaustion	[Wicksteed]	1894
	[Flux]	1894
Classifications of technical change	Hicks	1932
	Harrod	1937
	Solow	1963
Nash equilibrium	Nash	1951
Consumer surplus	Dupuit	1844
	Jenkin	1872
	Marshall	1890
Harberger triangles	Dupuit	1844
	Jenkin	1872
	Harberger	1964
Scitovsky curves	Scitovsky	1941

Table (continued)

Curve/diagram	Discoverer(s)	Date
Taxation of external costs	[Pigou]	1912
Monopoly	[Cournot]	1838
	Auspitz and Lieben	1880
	Marshall	1890
Discriminating monopoly	Robinson	1933
Duopoly	[Cournot]	1838
Monopolistic competition	Chamberlin	1933
	Robinson	1933
Kinked demand curves	Sweezy	1939
	Hall and Hitch	1939
Backward-bending labour supply curves	Robbins	1929
	Phelps Brown	1936
	Robinson	1937
Location theory	von Thünen	1826
	Launhardt	1885
	Lösch	1940
Hotelling's model	Hotelling	1929
The cobweb diagram	Ezekiel	1938
The Markowitz diagram	Markowitz	1952
Rent seeking diagrams	[Tullock]	1967
	Krueger	1974
	[Posner]	1975
Logistic growth curve	Verhulst	1838
	Pearl and Reed	1920
Graph theory	Economides	1996
General equilibrium analysis		
Circular flow diagrams	Quesnay	1758
	Knight	1932
	Samuelson	1948
The unit simplex	[Arrow and Debreu]	1954
Exchange of consumption box diagram	Edgeworth	1881
The role of numbers in competition	Edgeworth	1881

Table (continued)

Curve/diagram	Discoverer(s)	Date
Production possibilities frontier	Haberler	1930
	Viner	1931
	Leontief	1937
Utility possibility frontier	Allais	1943
	Samuelson	1947
Factor price frontier	Samuelson	1957
Pareto efficiency	Pareto	1906
Phase diagram	Marshall	1879
Theory of the second best and third best	Lipsey and Lancaster	1956
	McManus	1959
Offer curves	Marshall	1879
Stolper-Samuelson box	Lerner	1933
	Stolper and Samuelson	1941
Lerner diagram	Lerner	1933
Trade theory diagram	Leontief	1933
Four-quadrant diagram of the Heckscher-Ohlin model	Johnson	1957
Integrated world equilibrium	Dixit and Norman	1980
Optimum tariff	Edgeworth	1894
	Bickerdike	1906
Macroeconomics		
Keynesian income determination diagram	Samuelson	1948
Fleming-Mundell diagram	[Fleming]	1962
	Mundell	1963
Aggregate demand aggregate supply diagram	Dillard	1948
	Patinkin	1956
Phillips curve	Phillips	1948
UV or Beveridge curve	Dow and Dicks-Mireaux	1958

Table (continued)

Curve/diagram	Discoverer(s)	Date
Demand for money curve	Marshall Keynes	1871 1936
Non-neutrality of money	Keynes Ng Taylor McDonald and Solow	1936 1980 1980 1981
Laffer curve	Laffer	1981
Intertemporal utility maximisation	Fisher	1907
Solow-Swan model	Solow Swan	1956 1956
Lorenz curve	Lorenz	1905
Kuznets curve – income inequality	Kuznets	1955
Kuznets curve – environment	Grossman and Krueger	1991

Note: Square brackets indicate that the original presentation was algebraic.