1 Spatial econometrics
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1. OBSERVATION-LEVEL DATA DEPENDENCE

Spatial econometrics is a subset of the broader econometrics discipline that deals with modeling situations involving spatially dependent sample data. Regional scientists often work with regional data samples where each observation represents a single region and the sample observations consist of a collection of information for n different regions. A key assumption made by conventional linear regression models is that each observation is independent of other observations. Spatial regression models represent a set of methods that do not make this assumption, since it seems unrealistic to assume that economic and other types of activity taking place in one region are independent of those in neighboring regions.

Replacing the assumption of independence with one of spatial dependence between regions leads to a situation where spatial spillovers can arise. LeSage and Pace (2009) define spatial spillovers as non-zero cross-regional partial derivatives $(\partial y_j/\partial x_i \neq 0)$, so changes in characteristics x of region i can exert an influence on activity y taking place in other regions j. An important aspect of spatial econometric models is that they allow us to quantify spatial spillovers, which should be of interest to regional scientists.

A simple linear regression relationship between the exam scores of six students and the time each student spent studying shown in Table 1.1 could be viewed as a line-fitting exercise. Specifically, we wish to fit the linear relationship in (1.1), where study time serves as an explanatory variable and exam scores the dependent variable. Expression (1.2) uses more formal mathematical symbols, y, x, ε to represent exam scores, study time and disturbances or errors in the relationship (1.1).

$$\text{Score}_i = \alpha + \beta \cdot \text{Study time}_i + \text{error}_i, \quad i = 1,\dots,N \tag{1.1}$$

$$y_i = \alpha + \beta \cdot x_i + \varepsilon_i \tag{1.2}$$

Table 1.1   Exam scores and study time

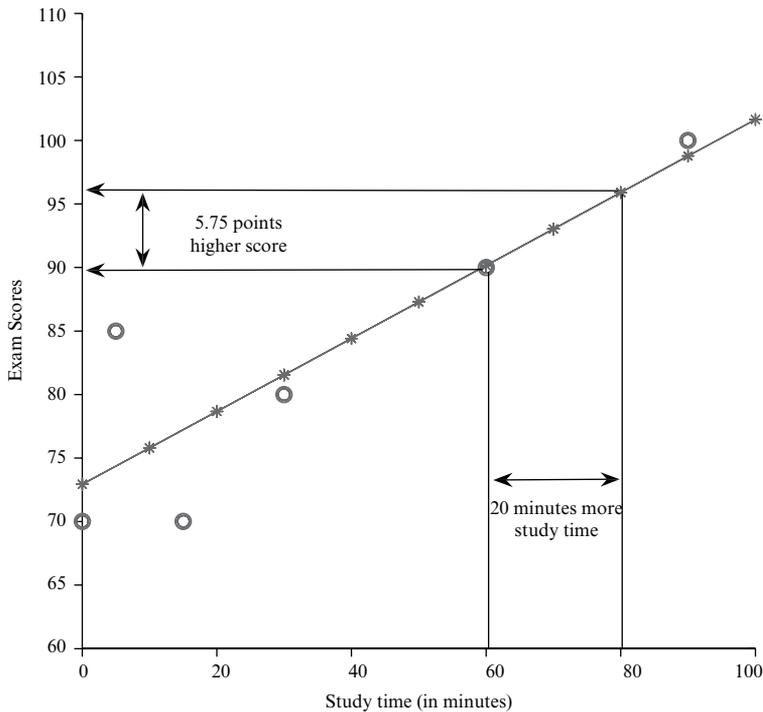| Student	Exam scores	Study times
John	$y_1 = 70$	$x_1 = 0$
Devon	$y_2 = 85$	$x_2 = 5$
Steve	$y_3 = 70$	$x_3 = 15$
Denise	$y_4 = 80$	$x_4 = 30$
Billy	$y_5 = 90$	$x_5 = 60$
Mary	$y_6 = 100$	$x_6 = 90$

Figure 1.1   Fitted line for exam scores and study time

The fitted line summarizes the relationship between the 12 numbers in Table 1.1 using two numbers, the intercept and slope. The intercept tells us the predicted exam score based on zero study time and the slope informs us how each additional minute of study time changes the exam score. From the fitted line shown in Figure 1.1, we see that an additional 20 minutes of study time would lead to a 5.75 points higher exam score. Parameter estimates for our model relationships allow us to: (1) learn about the nature of these relationships, for example how study influences exam scores; (2) make predictions about new observations not in our sample, e.g. predict the score of another student who spends 45 minutes studying; and (3) compare competing model relationships to see which better explains variation in the sample data information.

1.1   The Role of Independence

The linear regression used to estimate the intercept and slope of the fitted line was based on the usual assumption that each observation (student score) is independent of other students' scores. This means that a change in study time for a single student, say John, influences only his exam score, not the scores of other students.

   Statistical independence between observations is a simplifying assumption made by conventional regression models, which influences the formulae used to produce parameter estimates of the model (in our simple case, the intercept and slope are the model parameters). An important observation made by regional scientists is that the

Table 1.2   *Spatial proximity relations between students*

Seats occupied	John	Steve	Mary	Devon	Billy	Denise
Study time	0	15	90	5	60	30
Exam score	70	70	100	85	90	80

sample data information presented in Table 1.1 does not show us the relative locations of the six sample data observations, which would be the seats occupied by the students when taking the exam. This information is provided in Table 1.2, where we see that the students were seated in a single row, having one neighbor to the left and right, with the exception of two students located at the end of the row (John and Denise).

1.2   Spatial Dependence

If we examine the same sample data information arranged to reveal the spatial proximity of the observations/students taking the exam, we might question the assumption of independence between observations. Devon had a high exam score and spent only five minutes studying for the exam, but was seated between students Mary and Billy, who spent the most time studying and had the highest scores. Regional scientists who examine their sample data with respect to relative locations of the observations often find similar patterns in the data that cast suspicion on the assumption of independence used in conventional linear regression models.

1.3   Spatial Regression

It is relatively simple to modify the linear regression relationship to incorporate information regarding relative locations of the observations. The single-row spatial configuration of the students represents the simplest spatial structure, which can be encoded in a spatial weight matrix, as shown in (1.3). The first row of the matrix W shows that John (observation 1) has a single neighbor Steve (observation 2), while the second row (observation 2) records students (observations) seated (located) to the left and right of Steve. Each row of the spatial weight matrix W is row-normalized to have row-sums of unity. In the case of our observations (students), this results in observations with a left and right neighbor having two neighbors assigned values of 0.5, while observations located at the end of the row have weight matrix values of unity. The diagonal elements of W are zero to prevent an observation from being defined as a neighbor to itself.

The matrix W could be viewed an encoding the 'connectivity' relationship between our observations, specifically the locations of the students (observations) seated in a row relative to locations of other students (observations). A useful facet of connectivity matrices such as W is that powers of the matrix identify higher-order neighbors. Specifically, the matrix W^2 would identify observations that are neighbors to the neighboring observations by assigning these non-zero values. For our students, the second-order neighbors identified by W^2 would be students seated next to the students adjacent on the left or right. Using Mary as an example, the third row of the matrix W identifies Steve and Devon as left and right neighbors, while the third row of the matrix W^2 would contain

non-zero elements for John and Billy, who are second-order neighbors to Mary, that is neighbors to her neighbors. It would also contain a non-zero element for Mary, since she is a neighbor to her neighbors Steve and Devon, which makes her a second-order neighbor to herself, consistent with our definition. The matrix W^3 would continue in this fashion to identify third-order neighbors, those who are neighbors to the second-order neighbors John and Billy.

The $N \times N$ matrix W can be used to form a spatial lag of the dependent variable based on the matrix product Wy shown in (1.4). The spatial lag produces an average of the exam scores of neighboring students, and for students seated at the ends of the rows, the spatial lag reflects only the single neighboring student's score.

$$W = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{1.3}$$

$$Wy = \begin{pmatrix} y_1 \\ 0.5y_1 + 0.5y_3 \\ 0.5y_2 + 0.5y_4 \\ 0.5y_3 + 0.5y_5 \\ 0.5y_4 + 0.5y_6 \\ y_5 \end{pmatrix} \tag{1.4}$$

This spatial lag can be included as an explanatory variable in the regression model as shown in (1.5), where y is the $n \times 1$ vector of exam scores, ι_n is an $n \times 1$ vector of ones and α the associated intercept term, X is an $n \times 1$ vector of study times, our explanatory variable, with β the slope parameter. This spatial autoregressive model (SAR) indicates that we might expect to find that variation in exam scores of students depend on scores of neighboring students (those seated to the left and right). The scalar parameter ρ indicates the level or strength of dependence, and if this is not statistically significantly different from zero, then exams scores are independent. If ρ is positive and significantly different from zero, then exam scores on average over the sample are positively related to those of neighboring students, and for $\rho < 0$ inversely related.

$$y = \rho Wy + \alpha\iota_n + X\beta + \varepsilon \tag{1.5}$$

$$y = (I_n - \rho W)^{-1}(\alpha\iota_n + X\beta) + (I_n - \rho W)^{-1}\varepsilon \tag{1.6}$$

If there is no spatial dependence between exam scores ($\rho = 0$), the SAR model becomes a standard regression model that obeys the assumption of independence between exam scores. Estimates of ρ different from zero point to a situation where the relative location

of students matters in determining exam scores. It should be noted that there are alternatives to the SAR model spatial regression specification, details of which can be found in standard texts such as Anselin (1988) and LeSage and Pace (2009). The LeSage and Pace (2009) text includes details on maximum likelihood and Bayesian estimation of these alternative models. Kelejian and Prucha (1998) discuss two-stage least-squares estimation and Kelejian and Prucha (1999) provide a generalized method of moments estimation procedure.

1.4 Interpreting Spatial Regression Models

In the presence of spatial dependence between scores, changes in study time of one student can impact the exam scores of neighboring students, as well as neighbors to those neighboring students, and so on. This is because answers copied by Steve from Mary could in turn be copied by John from Steve's exam. To see that the model allows for this type of diffusion of study-time impacts, consider the partial derivatives associated with changes in the study-time explanatory variable, shown in (1.7) (LeSage and Pace, 2009).

$$\partial y/\partial X' = (I_n - \rho W)^{-1} I_n \beta \tag{1.7}$$

Expression (1.7) is an $n \times n$ matrix, since a change in a single student's study time could impact their own exam score, plus exam scores of all other students because of copying by neighbors, neighbors to neighbors and so on. This results in an $n \times 1$ vector of impacts from changing a single students' study time. We can however change each of the n students' study time in turn, leading to the $n \times n$ matrix of impacts. The main diagonal of the matrix represents own-partial derivatives ($\partial y_i/\partial x_i$), showing how a change in study time would directly impact each student's score, while the off-diagonal elements are cross-partial derivatives ($\partial y_j/\partial x_i$), showing the spatial spillover impacts on other students' scores arising from copying.

LeSage and Pace (2009) propose using an average of the main diagonal elements of the $n \times n$ matrix to produce a scalar summary measure they label 'direct effects'. This simplifies the task of interpreting estimates from the model, which take the form of an $n \times n$ matrix for each explanatory variable. This could be quite problematical if we were working with a relationship involving a sample of 3100 US counties. A scalar summary measure of the 'indirect effects' (spatial spillovers) is also proposed, based on the cumulative sum of the off-diagonal elements from each row, averaged over all rows. A scalar summary measure of the total impact of a change in study time on exam scores is the sum of the indirect plus indirect effects estimates.

To see how the SAR model allows for spillovers from neighbors to neighbors of neighbors, and so on, consider the infinite series expansion of the matrix inverse: $(I_n - \rho W)^{-1}$ shown in (1.8).

$$(I_n - \rho W)^{-1} = I_n + \rho W + \rho^2 W^2 + \rho^3 W^3 \ldots \tag{1.8}$$

The matrix W reflects left- and right-neighboring students to each student in our example, whereas the matrix W^2 points to students seated next to the neighbors, and W^3 are neighbors to the neighboring students, and so on. The scalar dependence parameter

ρ is upper-bounded by a value of one, so, for positive spatial dependence, $0 < \rho < 1$. This means that higher powers of ρ assign less weight to higher-order neighbors. An intuitively appealing aspect of this is that neighboring students would be able to copy answers most accurately, whereas copying by the next nearest neighbors might introduce some errors, diminishing the influence of increased study time by a single student on exam scores of more distant students as errors introduced by re-copying of answers occurs. In general, regional scientists believe that relationships are stronger between neighboring regions than those located far apart, so this aspect of the SAR model meshes nicely with regional science prior beliefs.

1.5   Software for Spatial Regression Models

Numerous software programs are available in the public domain for estimating spatial regression models that calculate the direct, indirect and total effects estimates. These routines also calculate and report *t*-statistics, which allow for statistical tests of significance of the direct and indirect or spillover effects associated with each explanatory variable. A set of freely available MATLAB functions for both econometric as well as spatial econometric analysis is provided by James LeSage at: www.spatial-econometrics.com, but users must purchase the MATLAB software program available at mathworks.com. An extensive manual documents the econometrics functions, referred to as a toolbox by The MathWorks Inc., and the spatial regression software functions reference chapters in LeSage and Pace (2009).

Programs (spdep) by Roger Bivand, written for the R-language, provide a software environment for statistical computing and graphics. The R-language software is an open-source, freeware product. More details are available on the R website. Roger Bivand, with the help of others, developed a suite of user-defined routines for spatial data analysis (sp) and spatial regression analysis (spdep, sphet), referred to as packages. The website on sp provides documentation for the functions in sp and spdep, and how to download the package.

A Stata module, SPPACK, for cross-section spatial-autoregressive models, by David M. Drukker, Hua Peng, Ingmar Prucha and Rafal Raciborski, requires users to purchase the Stata software program. Working papers that document the Stata commands are available at:

- http://econweb.umd.edu/ prucha/Papers/WP_spmat_2011.pdf,
- http://econweb.umd.edu/ prucha/Papers/WP_spreg_2011.pdf, and
- http://econweb.umd.edu/ prucha/Papers/WP_spivreg_2011.pdf.

2.   AN ILLUSTRATION OF SPATIAL REGRESSION

As an illustration of interpreting spatial regression model estimates, we use a dataset from Anselin (1988) on 49 Columbus, Ohio neighborhoods that relates crime rates (residential burglaries and vehicle thefts per 1000 households) to household income and housing values (both in thousands of dollars). *Ceteris paribus*, increases in household income levels in a single neighborhood (or *ceteris paribus*, increases in housing values)

Table 1.3   Spatial regression coefficient estimates

Variable	Coefficient	*t*-statistic	*t*-probability
Constant	44.8626	6.261	0.0000
Household income	−1.0264	−3.365	0.0007
Housing values	−0.2658	−3.005	0.0026
Wy	0.4349	3.708	0.0002

Table 1.4   Spatial regression effects estimates

Direct	Coefficient	*t*-statistic	*t*-probability
Household income	−1.0762	−3.605	0.0007
Housing values	−0.2839	−3.152	0.0027
Indirect	Coefficient	*t*-statistic	*t*-probability
Household income	−0.7664	−2.085	0.0423
Housing values	−0.2076	−1.748	0.0866
Total	Coefficient	*t*-statistic	*t*-probability
Household income	−1.8427	−3.299	0.0018
Housing value	−0.4916	−2.657	0.0106

should lead to a reduction in crime in the own neighborhood (the direct effect), and may also lead to reductions in crime in nearby neighborhoods (spatial spillover or indirect effects).

The maximum likelihood coefficient estimates for the parameters ρ and β are presented in Table 1.3, where we see spatial dependence in neighborhood crime rates, evidenced by the positive and significant coefficient (ρ) on the spatial lag of the dependent variable *Wy*. An important point is that the parameters β should not be interpreted as if they represented conventional regression coefficients depicting the impact of partial derivative changes in the explanatory variables on the dependent variable (crime rates). Spatial regression models that include spatial lags of the dependent variable are not amenable to such an interpretation.

The proper way to interpret the spatial regression model results is in terms of the effects estimates reported in Table 1.4 (see equation (1.7) and related discussion). We see negative (and significant) direct effects associated with changes in both neighborhood household income and housing values, suggesting that higher levels of these variables in neighborhood *i* lead to reductions in crime rates in neighborhood *i*. The scalar summary effects estimates are averaged over all 49 neighborhoods in the sample. Note that these estimates differ from the coefficients reported in Table 1.3 for household income and housing values, taking on slightly larger values. LeSage and Pace (2009) point out that some feedback effect comes into play in the direct effects estimates (see LeSage and Pace, 2009 for details). The direct effects estimates would be interpreted as indicating that a $1000 increase in household income in neighborhood *i* would lead to a reduction of 1.07 crime incidents per 1000 households. As in conventional regression model interpretation, this represents an average over the entire sample.

The indirect effects estimates reported in Table 1.4 show that spatial spillover effects from changes in household income in neighborhood i would lead to a cumulative decrease of 0.7664 crime incidents per 1000 households. The scalar summary indirect effect estimate cumulates spillovers falling on immediately neighboring regions, neighbors to these regions, neighbors to the neighbors of these regions, and so on. This should be clear from the infinite series expansion of the matrix inverse $(I_n - \rho W)^{-1}$ presented in (1.8). Of course, the magnitude of spillovers falling on immediate neighbors would be greater than those falling on more distant neighborhoods. If there is interest in partitioning the impacts falling on various orders of neighboring regions, LeSage and Pace (2009) show how to produce estimates of the profile revealing this decay of influence with orders of neighbors.

Spatial spillovers from changes taking place in neighborhood housing values are not statistically significant at the usual 95 percent level, but are significant at the 90 percent level. The total effects estimates reflect the sum of the direct plus indirect effects. From these we can see that ignoring spatial spillovers would lead to a drastic underestimate of the impact of household income and housing values on crime rates. The ability of spatial regression models to provide quantitative estimates of spillover magnitudes and to allow statistical tests for the significance of these represents a valuable contribution of spatial regression models to regional science.

3.   APPLICATIONS OF SPATIAL REGRESSION IN REGIONAL SCIENCE

Illustrative applications from the regional science literature that correctly interpret direct and indirect effects are briefly discussed. Readers should be warned that many past studies that appear in print do not correctly interpret the coefficient estimates from spatial regression models. Cross-sectional spatial regression applications are discussed in Section 3.1. A non-spatial application that considers peer group dependence in lieu of spatial dependence is discussed in Section 3.2. Models for network dependence are covered in Section 3.3 and for space-time panel data models in Section 3.4.

3.1   Cross-sectional Applications Involving Spatial Spillovers

Kirby and LeSage (2009) consider census tract level changes in commuting time between 1990 and 2000. They argue that: 'travel times to work from the 2000 Census showed an increase in average commuting times that is difficult to reconcile with the viewpoint expressed in earlier literature that suburbanization has provided a solution by acting as a traffic "safety valve", preventing a "traffic doomsday" from occurring in the face of urban growth'. In support of this contention, they cite statistics from Reschovsky (2004), indicating that in 1990, 12.5 percent of workers had travel to work times of 45 or more minutes, whereas in 2000 this figure was 15.4 percent, an increase of 10.39 percent. Kirby and LeSage (2009) use a spatial Durbin model (SDM) to examine factors that explain tract-level variation in the proportion of workers with long commute times. This model takes the form shown in (1.9).

$$y = \rho W y + \alpha \iota_n + X\beta + WX\theta + \varepsilon \tag{1.9}$$

This model includes a matrix of characteristics of neighboring census tracts (WX) as additional explanatory variables along with an associated parameter vector of θ. The motivation for these explanatory variables in the context of modeling long commuting times is that socio-economic demographic characteristics of persons living in neighboring census tracts should represent important explanatory variables. For example, if there is a large number of retired persons living in neighboring tracts, this should result in less congestion during commute-to-work times.

The motivation for spillovers or indirect effects in this setting is that commuting time congestion spillovers arise from shared roadways that traverse census tracts. Calculating direct and indirect effects for this model is slightly different from doing so for those associated with the SAR model, based on the expression in (1.10).

$$\partial y / \partial X' = (I_n - \rho W)^{-1} (I_n \beta + W\theta) \tag{1.10}$$

Kirby and LeSage consider changes in indirect effects over the 1990 to 2000 period, with an illustrative result being that employed females in 1990 impacted neighboring tract residents adversely, producing longer commute times. In contrast, for the year 2000, more employed females residing in a tract led to a decrease in commuting times for persons in neighboring tracts. They conclude (p. 470) that 'the predominance of statistically significant changes in demographic variables that reflect the age and gender distribution of Census tract residents points to changing demographic trends as a primary explanation for longer travel to work times'.

These findings are in contrast to those based on non-spatial regression models (Gordon et al., 2004, p. 16), which concluded that 'Higher income households may also be more likely to accept long commutes in order to consume the larger houses and more land available at more distant, especially exurban, locations.'

Another example of a cross-sectional regression that models local spatial spillovers is LeSage and Ha (2012). The distinction between local and global spatial spillovers is that spatial lags of the dependent variable of the type that exist in the SAR and SDM models imply global spatial spillovers. This is because the matrix inverse, $(I_n - \rho W)^{-1}$, which appears in the data generating process for these models (see expression (1.6)), has the infinite series expansion depicted in (1.8). This means that the partial derivatives shown in (1.7) allow for changes in a single region i to impact neighboring regions, neighbors to these neighboring regions, neighbors to the neighbors of these regions and so on, leading to global impacts or spillovers that spread out over the sample with decay of influence for higher-order neighbors. The decay of influence is evident when one considers that the parameter ρ takes values less than one, so higher powers represent smaller magnitudes.

Local spatial spillovers are those that fall only on neighboring regions identified by the matrix W, but not on higher-order neighboring regions. That is, we are restricting the spillovers to first-order neighbors. The spatial regression specification in (1.11) allows for local spatial spillovers to be modeled, and has been labeled the spatial lag of X (SLX) model by LeSage and Pace (2009). A related model is the spatial Durbin error model (SDEM) shown in (1.12) and (1.13), which models the disturbances as following a spatial autoregressive process. Since spatial autoregressive processes imply global spillovers, the

SLX and SDEM models imply local spillovers for changes in the explanatory variables, while the SDEM model allows for global spillovers in the disturbances.

$$y = X\beta + WX\gamma + \varepsilon \qquad (1.11)$$

$$y = X\beta + WX\gamma + u \qquad (1.12)$$

$$u = \lambda Wu + \varepsilon \qquad (1.13)$$

Interpretation of the SLX and SDEM models is quite simple, since the scalar summary own-partial derivatives reflecting direct effects are the coefficient estimates β, and a scalar summary for the cross-partial derivatives measuring spillovers is represented by the parameters γ. Furthermore, standard regression software can be used to estimate the SLX model, whereas specialized software would be needed for the SAR, SDM and SDEM specifications.

LeSage and Ha (2012, pp. 4–5) rely on such models to estimate the impact of in- and out-migration from nearby counties versus faraway counties on county-level social capital. They make an argument for delineating between nearby and faraway counties using a 40 mile cut-off. They state:

> The potential for county-level migration of population to impact social capital levels at particular locations in space arises from the conventional argument that social capital promotes trust and cooperation among agents. This trust increases socially efficient collective action, but requires investment in relationships and commitment of scarce resources to build social capital. It seems plausible that migration of population might have a negative impact on the stock of social capital by interfering with trust and cooperation among agents. However, it is also possible that migrants may take their social capital with them to new places of residence. In other words, the propensity to join social and civic organizations might reflect inherent traits of individuals who are willing to make commitments of their resources to building social capital in any community where they reside.

Consider the SLX model from (1.11), for simplicity. We can estimate the model using two matrices W_n to identify nearby neighboring regions and W_f faraway regions, as shown in (1.14). These matrices are used in conjunction with variable vectors M and O denoting in-migration and out-migration respectively.

$$y = O\beta_o + M\beta_m + W_n M\gamma_m + W_n O\gamma_o + W_f M\phi_m + W_f O\phi_o + \varepsilon \qquad (1.14)$$

Interpreting spillover estimates from this type of model is relatively straightforward. The partial derivative impacts associated with changes in out-migration O to own, nearby and faraway counties are β_o, γ_o, ϕ_o respectively. Similarly, we can easily test for significant impacts from in-migration M using the coefficient estimates β_m, γ_m, ϕ_m and associated t-statistics. Of course these scalar estimates average over all observations as in ordinary regression.

In many cases local spillovers should actually be the focus of interest, a point that has been overlooked in the spatial econometrics literature, where there seems to be a great deal of reliance on SAR and SDM models that logically imply global spillovers. For example, Lacombe (2004) uses a SAR model in conjunction with two matrices W_1, W_2,

representing neighboring counties within the state and across the state border. Interest centers on the effects of state-level variation in Aid to Families with Dependent Children (AFDC) and Food Stamp program benefit levels on female labor force participation. Changes in benefit levels of these two state-administered aid programs for low-income residents could have a spillover impact on counties in neighboring states, since it is possible for residents of border counties simply to move to neighboring states if there is a large discrepancy in aid benefits between neighboring states. This would seem to be a case where local spillovers might more accurately be the focus of interest. Would we really expect that changes in state-level AFDC and Food Stamp aid would lead to global spillovers? If so, changes in aid levels in Ohio could impact labor force participation in states neighboring Ohio, neighbors to the Ohio neighbors, neighbors to those neighbors, and so on. The implication is that changes in aid levels in Ohio would exert an impact on female labor market participation in states as distant as Maine and California.

3.2   Peer Group Dependence Applications

Blankmeyer et al. (2011) examine salaries of Texas nursing-home executives and argue that salary benchmarking practices should produce peer group dependence. They rely on an SDM model specification, but replace the spatial connectivity matrix W with a matrix that identifies peer nursing-home facilities, those having similar outlays on nursing services. They find evidence that nursing-home administrators' pay is significantly related to average pay of administrators in peer facilities.

Salary benchmarking is a prevalent practice by private as well as public organizations where salaries of comparable employees at peer (or similar) institutions are used to determine if adjustments are needed in compensation of employees. In practice, benchmarking on peer institutions will lead to something quite similar to simultaneous spatial dependence, only here the observational units are institutions rather than regions and the connectivity structure reflects 'neighboring' institutions rather than regions. Blankmeyer et al. (2011, p. 94) note:

> Given univariate or multivariate criteria of institutional similarity, conventional measures of univariate or multivariate distance (e.g. Euclidean or Mahalanobis distance) can be calculated. Peer institutions to each observation can be identified as those that are most similar, that is, those that exhibit smaller distances constructed using the similarity criterion. The result is an n by n matrix of (generalized) distances between each of the n observations and all others. To define peer groups each having m institutions, we would select the m nearest neighbors: the m most similar institutions.

This type of dependence leads to a situation where changes in explanatory variables (in Blankmeyer et al., 2011 these were characteristics of the nursing facilities) will impact compensation of own-institution administrators as well as that of administrators from other peer facilities. The impact on compensation of administrators from other facilities is of course an institutional spillover or indirect effect.

As an illustrative example of the type of results found by Blankmeyer et al. (2011), they establish that a manager who increases occupancy rates from the 25th percentile to the median would (other things equal) see a 19 percent increase in compensation, with the direct effect accounting for 8 percent of the increase and indirect effect 11 percent.

It should be noted that the scalar summary measures for direct and indirect effects set forth by LeSage and Pace (2009) cumulate spillovers across all peer institutions, plus peers to the peer institutions and so on. An implication of this is that spillovers falling on individual peers would be much smaller than the 11 percent cumulative indirect effects estimate. For example, Blankmeyer et al. (2011) rely on eight peer institutions determined using Bayesian model comparison methods. This would mean that even considering just immediate peers and ignoring peers to peers and higher-order peers, we would have (at most) $(1/8 \times 11\%) = 1.375\%$ magnitude of spillovers impact on the manager of a single peer institution. Non-spatial least-squares estimates, which are biased in the face of simultaneous dependence, indicated a smaller 16 percent compensation differential.

3.3   Network Dependence Applications

Flows across networks involving origins and destinations can also exhibit spatial dependence. Flows take many forms that have been extensively studied by regional scientists – population migration, commodity flows, traffic flows, knowledge flows – all of which reflect movements between origin and destination regions. LeSage and Pace (2008) define spatial dependence in this type of setting to mean that larger observed flows from an origin region A to a destination region Z are accompanied by: (1) larger flows from regions nearby the origin A to the destination Z, say regions B and C that are neighbors to region A, which they label origin-dependence; (2) larger flows from the origin region A to regions neighboring the destination region Z, say regions X and Y, which they label destination-dependence; and (3) flows from regions that are neighbors to the origin (B and C) to regions that are neighbors to the destination (X and Y), which they label origin-destination dependence.

Casual observation of traffic flows on a road network are consistent with this type of observation. If a large number of commuters starts at region A heading to destination region Z, we would expect to see congestion on tertiary roads near the origin, as well as congestion on traffic arteries near the destination Z. In addition, for the typical circular city we would expect that an origin suburb A has neighboring suburbs (B and C) and that a destination region in the central business district (CBD) Z has neighboring CBD regions (X and Y), with similar large flows from regions B and C to regions X and Y.

In the regional science literature the gravity model has been labeled a 'spatial interaction model' (Sen and Smith, 1995), because the regional interaction is directly proportional to the product of regional size measures. In the case of interregional commodity flows, the measure of regional size is typically gross regional product or regional income. The model predicts more interaction in the form of commodity flows between regions of similar (economic) size than regions dissimilar in size. In other contexts such as knowledge flows between regions (LeSage et al., 2007) the size measure of regions might be the stock of patents, so that regions with similar knowledge stocks would exhibit more spatial interaction taking the form of knowledge flows.

A regression model that has been labeled a 'gravity model' captures these notions, for example if one starts with the standard gravity model (cf. equation (6.4) in Sen and Smith, 1995) and applies a log transformation, the regression in (1.15) arises, where the vector y is an $n \times n$ flows matrix whose columns have been stacked to form an $n^2 \times 1$ vector.

$$y = \alpha \iota_{n^2} + X_d \beta_d + X_o \beta_o + \gamma g + \varepsilon \tag{1.15}$$

LeSage and Pace (2008) show that $X_d = I_n \otimes X$, where X is an $n \times k$ matrix of characteristics for the n regions, while $X_o = X \otimes I_n$, with $\otimes$ representing the Kronecker product. The vectors β_d and β_o are $k \times 1$ parameter vectors associated with the destination and origin region characteristics. If a log transformation were applied to the dependent variable y and explanatory variables matrix X, the coefficient estimates would reflect elasticity responses of OD flows to the various origin and destination characteristics. The scalar parameter γ reflects the effect of distance g, which is an $n \times n$ matrix of distances that has been stacked to form an $n^2 \times 1$ vector. The parameter α denotes the constant term parameter, and the $n^2 \times 1$ vector ε represents zero mean, constant variance, zero covariance disturbances.

Since the model in (1.15) is an ordinary regression, observations are assumed to be independent, so no spatial dependence is present. LeSage and Pace (2008) propose the model in (1.16) as a way to model spatial dependence of the three types set forth above, and LeSage and Fischer (2010) provide a more expository discussion of this model. This represents a spatial autoregressive (SAR) extension of the non-spatial model in (1.15), which can be viewed as filtering for spatial dependence related to the destination and origin regions.

$$(I_N - \rho_d W_d)(I_N - \rho_o W_o)y = \alpha \iota_N + X_d \beta_d + X_o \beta_o + \gamma g + \varepsilon \tag{1.16}$$

The $n^2 \times n^2$ matrix W_d is constructed from the typical row-stochastic $n \times n$ matrix W that describes spatial connectivity between the n regions. The matrices W_d and W_o can be written using the Kronecker products: $W_d = I_n \otimes W$, and $W_o = W \otimes I_n$, where I_n is an identity matrix of order n and we note that $W_d \times W_o = W \otimes W$. The resulting model from LeSage and Pace (2008) takes the form of (1.17), where the scalar parameter ρ_d measures the strength of destination-dependence, ρ_o captures origin-dependence, and ρ_w origin-destination dependence.

$$y = \rho_d W_d y + \rho_o W_o y + \rho_w W_w y + \alpha \iota_{n^2} + X_d \beta_d + X_o \beta_o + \gamma g + \varepsilon \tag{1.17}$$

This type of model produces global spatial spillovers and requires proper interpretation of the own and cross-partial derivatives that show how changes in regional characteristics X impact flows between origin and destination regions. Behrens et al. (2012) provide an economic justification for this type of model using a theoretical model for international trade flows. As in the SAR model, the coefficients β_d, β_o do not correspond to the usual regression interpretation, as can be seen from the partial derivatives for this model. These would take the form of an $n^2 \times n^2$ matrix, shown in (1.18).

$$\partial y / \partial X' = (I_n^2 - \rho_d W_d - \rho_o W_o y - \rho_w W_w)^{-1}(I_n^2 \beta_d + I_n^2 \beta_o) \tag{1.18}$$

LeSage and Thomas-Agnan (2014) elaborate on interpretation of this model is beyond the scope of this description, but the intuition for the large number of changes that arise from changing the characteristics of a single region i is that flows across the entire network will be affected. For example, making a single region i more attractive as a

destination will produce a host of impacts on flows to and from all other regions in the network. For example, since region i is more attractive as a destination, this will increase inflows to region i from all other n origins and reduce outflows from region i to all other n destinations. Spatial dependence also implies an increase of inflows to regions neighboring i and a decrease in outflows from neighbors to region i.

One way to think of this is to consider the impact of a traffic accident on a road network. The resistance/congestion created by the accident produces impacts on all of the tertiary roads near the road segment affected by the accident. It also inhibits flows along the road segment with the accident that originate far away and end far from the accident, but require passing through the road segment affected by it.

3.4   Space-time Dependence Applications

In a panel data setting where observations are available for a cross-section of N regions over T time periods (a panel data set), there is the possibility of both space and time dependence, as well as covariance between spatial and time dependence. This type of dependence can reside in either the dependent variable or the model disturbances. Parent and LeSage (2010, 2011, 2012) propose a 'space-time filter' shown in (1.19) as a way of modeling these types of dependence. The filter relies on a Kronecker product of the matrices C and B, shown in (1.19), where L is a $(T + 1) \times (T + 1)$ matrix representing the time-lag operator.

$$C \otimes B = I_{N \times (T+1)} - \rho I_{T+1} \otimes W - \phi L \otimes I_N + (\rho \times \phi)L \otimes W,$$

$$C = \begin{pmatrix} \psi & 0 & \dots & 0 \\ -\phi & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & -\phi & 1 \end{pmatrix} \quad B = (I_N - \rho W) \tag{1.19}$$

This filter implies a restriction that the parameter associated with spatial effects from the previous period $(L \otimes W)$ is equal to $-\rho \times \phi$. Parent and LeSage (2010) show that applying this space-time filter to the error terms greatly simplifies the estimation procedure for a model where space-time dependence is modeled in the disturbance structure.

The same filter can be applied to the dependent variable vector in a panel data model to produce a space-time dynamic panel data specification, shown in (1.20).

$$(C \otimes B)y = \iota_{N \times (T+1)}\alpha + x\beta + \eta \quad \eta \sim N(0, \tilde{\Omega}) \tag{1.20}$$

where $y = (y_0', \dots, y_T')'$ is an $N \times (T + 1)$ column vector, $x = (x_0', \dots, x_T')'$ is an $N(T + 1) \times k$ matrix and $\tilde{\Omega} = \sigma_\mu^2(J_{T+1} \otimes I_N) + \sigma_\varepsilon^2 I_{N \times (T+1)}$, with $J_{T+1} = \iota_{T+1}\iota_{T+1}'$.

The resulting model specification can be written as in (1.21) for each time period $t = 1, \dots, T$, where the restriction on the parameter $-\rho \times \phi$ associated with $(L \otimes W)$ is relaxed by introducing a free parameter θ to be estimated.

$$y_t = \rho W y_t + \phi y_{t-1} + \theta W y_{t-1} + \iota_N \alpha + x_t \beta + \eta_t \quad \eta_t = \mu + \varepsilon_t \tag{1.21}$$

Debarsy et al. (2012) show that the data-generating process for this type of model specification can be expressed using matrix/vector notation as shown in (1.22), based on some assumptions regarding treatment of the initial period observation. They condition on the initial period observation and assume that this period is subject only to spatial dependence, which implies a dependent variable vector for the entire sample: $Y = (y_1', \ldots, y_T')'$. Given this treatment of the initial period, the model can be expressed by replacing the $NT \times N(T + 1)$ space-time filter $C \otimes B$ by the $NT \times NT$ matrix Q shown in (1.25).

$$Y = Q^{-1}[\iota_{NT}\alpha + X\beta + \mu + \varepsilon] \tag{1.22}$$

$$Y = \sum_{r=1}^{K} Q^{-1}(I_{NT}\beta_r)X^{(r)} + Q^{-1}[\iota_{NT}\alpha + \mu + \varepsilon] \tag{1.23}$$

$$Q = \begin{pmatrix} B & 0 & \cdots & & 0 \\ C & B & & & 0 \\ 0 & C & \ddots & & \vdots \\ \vdots & & \ddots & & \\ 0 & \cdots & & C & B \end{pmatrix} \quad C = -(\phi I_N + \theta W) \quad B = (I_N - \rho W) \tag{1.24}$$

Expression (1.23) expresses the model using $X^{(r)}$ to denote the rth column (explanatory variable) from the $NT \times K$ matrix X, allowing us to express this DGP in a form suitable for considering the partial derivative impacts that arise from changes in the rth explanatory variable. Debarsy et al. (2012) note that the matrix Q^{-1} takes the form of a lower-triangular block matrix, containing blocks with $N \times N$ matrices as shown in (1.25).

$$Q^{-1} = \begin{pmatrix} B^{-1} & 0 & \cdots & & 0 \\ D_1 & & & & \vdots \\ D_2 & D_1 & \ddots & & \\ \ddots & & \ddots & & 0 \\ D_{T-1} & D_{T-2} & \cdots & D_1 & B^{-1} \end{pmatrix} \quad D_s = (-1)^s (B^{-1}C)^s B^{-1}, \quad s = 0, \ldots, T - 1 \tag{1.25}$$

This greatly simplifies calculating the partial-derivative impacts for this type of model. Debarsy et al. (2012) show that the one-period-ahead impact of a permanent change in the rth variable at time t can be expressed as in (1.26), where a permanent change means that the explanatory variable values increase to a new level and remain there in future time periods.

$$\partial Y_{t+1}/\partial X^{r'} = (D_1 + B^{-1})[I_N\beta_r] \tag{1.26}$$

By analogy with our cross-sectional discussion of the scalar summary measures proposed by LeSage and Pace (2009) for the SAR model, the main diagonal elements

of the $N \times N$ matrix in (1.26) for the one-period time horizon represent (cumulative) own-region impacts that arise from both time and spatial dependence. The sum of off-diagonal elements of this matrix reflects spillovers measuring both contemporaneous cross-partial derivatives and diffusion impacts, reflecting the spread over space with the passage of time.

An application involving the impact of highway expenditures on commuting times from Parent and LeSage (2010) allows us to discuss interpretation of this type of model. An increase in expenditures on highways in region i during period t should lower commuting times on the improved road segments in region i during future time periods, which is captured by the dynamic time-dependence part of the panel data model. These expenditures/improvements for roads in region i should also improve commuting times on neighboring road segments that extend into neighboring regions j, and this is captured by the spatial dependence (spillovers) part of the model. Commuters in neighboring regions j will adjust their behavior in future time periods to take advantage of highway improvements in region i, which is captured by the space-time covariance or diffusion aspects of the model. These various aspects of changes in commuting times arising from highway improvements are captured by the partial derivatives for this model. If we let y_{it} denote commuting times in region i at time t and x_{it} highway improvement expenditures, the own-partial derivatives, measured by $\partial y_{it+T}/\partial x_{it}$, reflect T-horizon own-region i changes in commuting times response to changes in region i highway improvements. Cross-partial derivatives measured by $\partial y_{jt}/\partial x_{it}$ reflect current period responses of commuting times in neighboring regions j, or contemporaneous spatial spillovers. Cross-partial derivatives $\partial y_{jt+T}/\partial x_{it}$ measure future period (T-horizon) changes in commuting times in neighboring regions j due to highway improvements made in region i at time t. These reflect diffusion impacts over time that arise as commuters adjust their behavior.

The applied illustration in Parent and LeSage (2010) shows a situation where it is possible to separate out the time versus spatial impacts because the model relies on the restriction $\theta = -\phi\rho$, which results in a separable space-time covariance structure. In contrast, the applied illustration in Debarsy et al. (2012) does not allow for separation of these two types of impacts. Nonetheless, the own-partial derivatives provide empirical estimates of the own-region impacts while cross-partial derivatives can be used to quantify the combination of spatial spillovers and space-time diffusion impacts.

4.   CHAPTER SUMMARY

Regional scientists frequently investigate situations where changes made in characteristics of one region impact outcomes in the own and neighboring regions. Past applications of spatial regression models have misinterpreted the coefficient estimates from these models, which has obscured a valuable contribution of spatial econometrics for applied regional science research (LeSage and Pace, 2009). Spatial regression models have the ability to provide estimates of own- and other-region impacts that arise from changing characteristics of a single region. One aspect of these models that has hindered proper interpretation is the fact that the partial-derivative impacts take the form of an $n \times n$ matrix, where n is the number of regions being analyzed. LeSage and Pace (2009) reduce

the $n \times n$ matrix of partial derivatives using scalar summary measures that produce: (1) a single coefficient reflecting the average own-region impacts; (2) a single coefficient measuring the average cumulative other-region (spatial spillover) impacts; and (3) a single coefficient for the total impact, which is the sum of the direct and indirect impacts. In addition, they provide a method for calculating *t*-statistics that can be used to draw inferences regarding the significance of the scalar summary measures for the various types of impacts.

This approach to interpreting spatial regression models can be extended to simpler models involving only spatial lags of the explanatory variables that can be estimated using ordinary regression software (LeSage and Ha, 2012). It is also possible to extend the method to spatial interaction or gravity models (see Chapter 8 in LeSage and Pace, 2009), to dynamic space-time panel models (Parent and LeSage, 2010 and Debarsy et al., 2012) and to spatial probit and Tobit regression models (LeSage et al., 2011).

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