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1. Reflections on entropy, complexity and spatial dynamics: the rebirth of theory?

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1. CONCEPTUAL OVERVIEW

1.1 Prologue

‘Over the last three decades, the sheer scale and scope of global interconnectedness has become increasingly evident in every sphere, from the economic to the cultural. Worldwide economic integration has intensified as the expansion of global commerce, finance, and production binds together the economic fortunes of nations, communities, and households across the world’s major trading regions and beyond within an emerging global market economy’ (McGrew, 2020, p. 16). *Globalization* generally refers to the rapidly expanding network of interconnections that characterize modern socioeconomic life – in essence, complex connectivity (Tomlinson, 2013). Central to and helping to drive these structural economic transformations and growing complexity are contemporary informatics technologies coupled with transportation infrastructure and systems, which have intensified and expedited the flow of people, goods and information, contributing to greater interaction, connectivity and cohesion, locally and globally. In this context, shocks of various magnitudes and types, such as pandemics, earthquakes, hurricanes, economic recessions and geopolitical events, have been shown to produce uncertainty and dramatic change at all socioeconomic and spatial scales – from neighborhoods to the entire globe (Broitman and Czamanski, 2021a).

In recent years our ability to model complex spatial dynamical systems has expanded enormously. In this regard we have witnessed new developments and significant progress in advanced modeling efforts (Nagurney and Qiang, 2009; Reggiani and Nijkamp, 2006, 2009). Now, in the era of big data, driven by the diffusion of information technology, the proliferation of human and machine sensors, and rapidly expanding capacities to process, store, disseminate and analyze data, there are new opportunities and potentialities for modeling complex, dynamical spatial systems (Schintler, 2020; Schintler and Chen, 2017; Schintler and Fischer, 2020). Simultaneously, there are serious concerns that big data may signal the end of theory, as it encourages a shift from the conventional hypothesis-driven scientific method to the use of data-driven techniques that seek to uncover patterns and correlations in the absence of any guiding hypotheses (Anderson, 2008).

Such trends and developments necessitate a conceptual transition in our thinking, requiring new paradigms and calling for new research imperatives regarding our approaches to understanding and explaining complex spatial systems. Indeed, it is time to consider whether fundamental concepts and theories – and, consequently, related models – underlying spatial economic science are still valid and useful, especially taking into account contemporary trends and transformations. Along these lines, it is also essential to explore whether there is a need

for innovative theoretical trajectories, particularly to better understand and predict complex spatial phenomena in our increasingly interdependent, dynamic and uncertain world. In these respects, a first step is to look concisely at the history of the foundational models, theories and concepts underpinning spatial economic analysis.

1.2 From Entropy to Complexity in Spatial Economics

Spatial economics aims to identify and understand the factors and dynamics governing the location and distribution of economic activities in space, and consequently the spatiotemporal evolution of socioeconomic systems. Accordingly, the space-economy can be characterized as a complex economic system enhanced and complemented by the dimension of space. In spatial economics, the most fundamental models in order of their historical appearance are as follows (Reggiani, 2021):

1. The rank–size rule/Zipf’s law (Zipf, 1949)
2. Gravity models (Isard, 1956)
3. Spatial interaction models (SIMs) (Wilson, 1970)
4. Discrete choice models (McFadden, 1973)

Interestingly, these four types of models are analytically and mathematically compatible. For example, SIMs are the ‘focal’ model emerging from gravity theory (Isard, 1956); they also derive from statistical information principles and entropy maximization (Wilson, 1967; 1970; 2021, in this volume). Furthermore, SIMs can be formulated according to the rank–size rule (Batty, 2010). The rank–size rule, in fact, like SIMs, can be derived from an entropy maximization approach, specifically by maximizing the most likely distribution of the population (Batty, 2021, in this volume; Reggiani and Nijkamp, 2015). Additionally, SIMs are linked to logit models and discrete choice models based on microeconomic theories, including random utility maximization (Erlander and Stewart, 1990; Fotheringham and O’Kelly, 1989; Reggiani, 2021).

SIMs have a solid theoretical foundation that allows for robust and precise modeling of complex spatial economic systems; also, entropy and utility theories suggest that a unified view may be plausible.

Entropy is considered by Wilson to be the most likely macro-distribution for micro-level interactions in a spatial economic system from which a SIM emerges as an equilibrium solution. Building on Wilson’s foundational work, entropy in an urban and regional context has been subsequently interpreted as a measure of interactivity (Erlander and Stewart, 1990), but also of uncertainty – that is, as the most probable distribution with the greatest number of microstates giving rise to it (Nijkamp and Reggiani, 1992). Thus, entropy can be described as ‘a probabilistic concept describing the outcome of a stochastic spatial allocation process’ (Nijkamp and Reggiani, 1992, p. 18).

Alternate theoretical conceptualizations of entropy and new entropy modeling frameworks emerged in the 1970s. In particular, the interpretation of entropy in a utility approach (Hansen, 1972) and its use in the programming (optimization) models (Nijkamp, 1975), as well as in the minimum information principle (Snickars and Weibull, 1977), came to the fore (for details, see Nijkamp and Reggiani, 1992, as well as Section 2.1). During this period, scholarly exchanges and dialogues on the possible behavioral-economic interpretations and applications of entropy

in spatial economics were intense, with the primary motivation being to develop a unified theoretical framework based on the concept.

In the 1980s there was burgeoning interest in how exogenous fluctuations and random noise drive systems far from equilibrium and thus towards the stochastic processes, and consequently new interest in the use of models with probabilities of transitioning from one state to another, such as the master equation approaches (Haag, 2021, in this volume; Nicolis and Prigogine, 1977; Weidlich and Haag, 1983), compartmental analysis (De Palma and Lefèvre, 1987) and optimal control for stochastic multistage, multi-actor choice processes (Leonardi, 1983). These new research endeavors led to dynamic analyses of entropy, building on the work of Harris and Wilson (1978): see, for example, the use of stochastic optimal control models, taking into account cumulative entropy functions (Nijkamp and Reggiani, 1988; Benguigui et al., 2000; Benguigui et al., 2004).

Interest in dynamic frameworks and concepts continued into the 1990s, at which time the focus was on ecologically based theories, catastrophe/bifurcation theories and chaos theory embedding endogenous fluctuations. The primary thrust of this research was on (in)stability and (dis)equilibrium in socioeconomic systems and thus on complexity issues and considerations (see again, for a review, Nijkamp and Reggiani, 1992). In the last few decades, the role of connectivity as a critical element of complex systems and its implications for the analysis of complex spatial economic networks have received a great deal of attention (Patuelli et al., 2010; Reggiani and Schintler, 2005). In spatial economics, spatial networks can be interpreted as connectivity structure for spatial communication and transportation systems, comprising primary nodes (hubs) which act as receivers and senders (i.e. centers with push and pull forces) connected by edges (i.e. channels or corridors) (Nijkamp and Reggiani, 1998). The richness of scientific contributions in this area is highlighted in the seminal articles of Simon (1962) and Casti (1979) and, more recently, in the work by Barabási and his group (Barabási, 2005) and other scholars in network science (see, among others, Barthélemy, 2011; Boccaletti et al., 2006; Caldarelli and Vespignani, 2007; Scott, 2017; Vega-Redondo, 2007; Vervest et al., 2009).

Reflections on entropy, including efforts to apply the concept in new and different ways, have continued (Haynes et al., 2021, in this volume). Moreover, there are ongoing references to how entropy and complexity are connected (Batty, 2021; Berry, 2021; Rosser, 2021, in this volume). However, interest has diminished in developing entropy as a ‘fundamental and unifying theoretical framework’ for modeling and understanding complex spatial economic systems, that is, the space-economy.

In this era of uncertainty, global connectivity and socioeconomic disruption, it is time to revisit the possibility of entropy as a concept and theoretical construct that can unify the study of complex spatial economic systems. To this end, some essential research questions are as follows:

1. What is the current state of research and knowledge on complexity and spatial dynamics, including the role of entropy in this regard?
2. Are there conceptual, theoretical and methodological advances – complementary or alternative to entropy – that can provide new insights and research directions in complexity and spatial dynamics?
3. Can entropy be considered ‘the universal principle’ that enables us to join probabilistic approaches with behavioral approaches to understand, model and predict spatial economic systems, capturing uncertainties and discontinuities in their evolution – considering complexity?

Given these questions, an endeavor involving a synthesis of knowledge from different scientific traditions and perspectives is paramount, especially considering new theories and methodologies and emerging discussions in disciplines from mathematics to physics to economics to geography. In light of all this, this handbook aims to explore spatial dynamical systems from fundamental and cross-disciplinary perspectives, to exploit diverse, foundational and innovative perspectives and approaches, focusing on entropy and complexity as guiding and integrating concepts. As a starting point for methodological and theoretical reflections, we first present some background on the three thematic pillars of the handbook: entropy, complexity and spatial dynamics (Section 2). We then provide a synthesis of novel empirical, methodological and theoretical insights and lessons from the chapters in this volume (Sections 3 and 4).

2. SOME REFLECTIONS ON ENTROPY, COMPLEXITY AND SPATIAL DYNAMICS

2.1 Entropy

The concept of entropy has a long history. First conceived in the nineteenth century, it was understood at that time as the Second Law of Thermodynamics, stating that closed systems always progress in the direction of increasing disorder (or entropy). By the twentieth century, the concept of entropy had evolved and assumed more general forms, especially given Shannon's foundational work to position the concept in information theory and statistical mechanics (Shannon, 1948) and the development of the entropy maximization principle by Jaynes (1957).

Entropy was used for the first time in an urban and regional context in the middle of the twentieth century (Wilson, 1967; 1970). In particular, Wilson saw that entropy maximization in cities and regions with many participants was about the most probable distribution of activity flows. Thus, entropy maximization could be used to model spatial interaction. While Wilson's first application of entropy maximization was to commuting flows, he later expanded and modified the framework in various ways, leading to an entire family of spatial interaction models. It is important to note that Wilson intended such developments to be 'independent of physics'. He states, 'There is no analogy involved but simply the application of a superconcept, entropy, which is applicable in a variety of disciplines' (Wilson, 2021, p. 25, in this volume).

Over the years and following the work of Wilson, urban and regional applications and conceptualizations of entropy have expanded enormously (see also Section 1.2).

First, there have also been efforts to identify and establish consistencies between entropy-based approaches with micro- and macro- (disaggregate and aggregate) economic frameworks. The reconciliation and unification of the entropy-maximizing and the random utility-maximizing methods have been proposed by several authors (Anas, 1983; Batten and Boyce, 1986; Choukroun, 1975; Smith, 1978, 1984). In this regard, it has been argued that although random utility-maximizing models are estimated from disaggregated data and entropy-maximizing models from aggregate data, it is perfectly reasonable and computationally feasible to recast the latter in a disaggregate form or the former in an aggregate one (Batten and Boyce, 1986; Fischer and Reggiani, 2004). Thus, the entropy-maximizing and utility-based approaches can be considered complementary views that may lead to similar (and even identical) parameter

estimation equations and solutions for the same problem: ‘Furthermore, the same model without any aggregation error may be derived in disaggregate form from both entropy and utility maximization and can therefore be estimated’ (Batten and Boyce, 1986, p. 378).

Second, entropy has been considered in a dynamic setting (Nijkamp and Reggiani, 1992) and linked to urban economics using Zipf’s law (1949). Regarding the latter, it has been shown that the rank-size rule/Zipf’s law is closely associated with entropy maximization, where the related power (or exponential) function depends on the form of the analytical constraint and the rank has an economic interpretation, that is, a shadow price (Reggiani and Nijkamp, 2015). Scholars continue to argue about the existence of power laws in various kinds of urban and regional complex systems and, more generally, the role of entropy in spatial complexity.

The idea that spatial entropy is a measure of complexity in spatial systems has also been developed (Batty, 2005; 2021, in this volume; Song et al., 2010; Wilson, 2000). Specifically, entropy measures disorder/order, uncertainty/certainty, dispersion/agglomeration, disorganization/organization and diversity/concentration. Accordingly, entropy is a critical concept in studies of resilience, sustainability, inequality and segregation (Rose and Dormady, 2021, in this volume). Entropy is also viewed as a process, shaping and shaped by the evolution of complex spatial systems (Batty, 2021, in this volume). Indeed, cities, regions and spatial systems more generally exhibit complex patterns as they grow and progress, in which various combinations of and interactions between fundamental units generate system dynamics. Moreover, complex systems display various qualitative patterns of behavior and phases that result from different forms of internal organization. Thus, the study of complexity is a search for the principles that generate self-organized, emergent phenomena – where entropy has a critical role to play. Interestingly, the term ‘complex’ derives from the Latin word *complexus*, meaning ‘twisted together’; thus, the concept embeds the assemblage of and dynamic interrelations between different units and elements: the concept of network, which also underlies spatial entropy (Reggiani, 2021).

However, while entropy has some well-defined interpretations and concrete formalizations, complexity has turned out to be more challenging to define and conceptualize. Additionally, the formal linkages between entropy and complexity remain murky. Therefore, a significant research challenge is understanding such ties and relationships (especially the ‘formal’ dynamic micro-macro relationships). To help serve as a basis for such explorations, we need a basic understanding of complexity, which is provided in the next section.

2.2 Complexity

In this increasingly globalized and networked era, the concept of complexity – and complex systems – has received a great deal of attention. While there are many definitions of complexity, John Casti captures the essence of the concept elegantly and compellingly: ‘The primary idea of complexity concerns the mapping of a system’s non-intuitive behavior, particularly the evolutionary patterns of connections among interacting components of a system whose long-run behavior is hard to predict’ (Casti, 1979, p. 97).

Casti further delineates between static and dynamic complexity (Reggiani, 2021, p. 327–8):

Static complexity: refers to the network configuration, where the components are put together in an interrelated and intricate way. Network configuration

concerns, for example, the number and type of hierarchical structures, the type of connectivity patterns, the variety of components, and the strength of the interactions. Clearly, static complexity can be roughly measured by the above-mentioned variables.

Dynamic complexity: concerns the dynamic network behavior governed by nonlinearities in the interacting components. Here, two rough measures can be the computational complexity and the evolutionary complexity. The latter measure can be carried out by means of appropriate nonlinear models, like chaos models, in particular, and evolutionary models in general, which are able to map out the dynamic (random) network patterns.

Regarding dynamic complexity from a behavioral perspective, Couclelis (2009) introduces the concept of ‘polyplexity’, which accounts for space and time underlying and embodying certain phenomena, dimensions which can themselves be complex. In this context, Couclelis proposes a strategy for modeling complex systems based on three interrelated components: the system of interest itself, the environment in which the system is positioned and the spatiotemporal features and context of the system.

In this vein, a basic view of complexity was provided by the Nobel Laureate Herbert Simon, as follows: ‘Roughly, by a complex system, I mean one made up of a large number of parts that interact in a non-simple way. In such systems, the whole is more than the sum of the parts . . .’ (Simon, 1962, p. 468). Therefore, a complex system is also one in which elements and subsystems interact in intricate and unpredictable ways. At the same time, complex systems have an underlying architecture guided by universal principles, including the dynamic properties of hierarchically organized systems. In other words, there are hierarchies and stratifications in complex dynamical systems that greatly simplify their behavior (Simon, 1962). Such ‘simple’ characteristics have been subsequently articulated from a network perspective by Albert L. Barabási, who argues that there are universal properties in complex network topologies, thus linking network theory to complexity theory (Barabási, 2007; 2017).

Consequently, universal properties in complex hierarchical and networked systems enable us to analyze and measure the static complexity; however, our ability to capture the inherent (dynamic) behavioral complexity remains a challenge. Nevertheless, both system theory and spatial economic theory can help clarify the dynamic complexity of spatial economic phenomena by exploiting the fundamental rules and properties of such systems (Reggiani, 2021). One ‘simple’ and general principle that cuts across different complex spatial economic systems is certainly entropy (Batty, 1974; Simon, 1962; Wilson, 1970).

In this current methodological debate on complexity versus regularity/simplicity, an intriguing research question is whether ‘real-world’ complex spatial systems, such as cities and regions, have an underlying structure guided by simple – socioeconomic – rules or principles (Reggiani and Nijkamp, 2009). Indeed, cities and regions exhibit complex economic processes that shape and are shaped by an assortment of factors and forces (for example, causal mechanisms that drive regional disparities and inequalities in productivity and economic growth and development). In this regard, it is crucial to distinguish between the principal regularities in urban and regional growth and development and the events and processes that are not spatially or temporally stable or regular and that thus affect the development pathways in uncertain and even irreversible ways (Storper, 2013). To this end,

‘spatial dynamics’ analysis can be especially fruitful if we adopt an interdisciplinary approach, exploiting the diversity of perspectives and scholarly contributions across different disciplines and fields (such as socioeconomic and physical sciences), as advocated by Michael Storper (2013, p. 5): ‘Urban and regional is a noisy and complex phenomenon. Its most significant causes cannot be understood through the tools of any single discipline or theory, even the “economic” ones.’

Given that dynamics are essential and inherent features of complex systems and that entropy is an outcome of dynamic processes and a process itself, we turn next to some reflections on spatial dynamics.

2.3 Spatial Dynamics

The concept of spatial dynamics, as it relates to complex spatial socioeconomic systems, emphasizes evolutionary processes and emergent behavior in space. In the fundamental work by Isard and Liossatos (1979) and Liossatos (1980), spatial dynamics is conceptualized as the spatiotemporal dimension of socioeconomic activities and related transformations in spatial form and structure. In this context, an evolutionary perspective in the study of complex spatial systems, accounting for instabilities and fluctuations in systems (for example, vis-à-vis dissipative structures, structural (in)stability and linkages with entropy) has been advocated (Liossatos, 1980).

Cities and regions are prime examples of complex spatial dynamical systems. Indeed, as pointed out by Liossatos, the growth and decline of regions is an expression of spatial dynamical processes. Spatial dynamics have also been tied to innovation processes, as well as to urban centralization and decentralization and the role of cities in facilitating spatial dynamics (Nijkamp, 1983). Consistent with previous definitions, Irwin (2010) suggests that spatial dynamics imply a transitional process of change in spatial patterns over time, as captured by urban economic models and simulation-based models, such as agent-based models (Irwin, 2010). Desmet and Rossi-Hansberg (2010) emphasize the necessity of developing a dynamic spatial theory. By considering the links between growth and regional/urban economics, they formulate a framework for capturing the linkages between spatial and temporal dimensions. An overview of spatial dynamics, considering different assumptions and implications regarding space–time conditions (e.g. discrete versus continuous), along with conceptual, methodological and technical issues surrounding the methods of space–time data analysis, is provided by Rey (2021).

Even in the eighteenth century, economists like Adam Smith (1776) discussed the role of regions and cities and their functions in an economic context (Mills, 1986). However, spatial economics did not emerge as an accepted discipline until the middle of the twentieth century. Since then, regional and urban economic theorists have made ‘formidable progress in achieving a further understanding of the structure and evolution of spatial economic systems’ (Nijkamp, 1986, p. 1). A principal motivation of spatial economics is to explain dynamic processes in cities and regions, as well as across systems of settlements. In this regard, there are many important questions. For example, why do cities exist in the first place? What forces contribute to their emergence and evolution? Does agglomeration play a central role? How do local spillovers and agglomeration processes explain how cities and regions facilitate the movements of goods, people and ideas? Why are settlements not of equal size, despite equilibrating forces, and alternatively why is there not a single city where everyone in the world resides?

Until recently, the vast majority of studies concerned with cities and regions assumed that such systems tend towards equilibrium. Studies of wealth inequality generally assume that an observed wealth distribution reflects a system in equilibrium. Even after the monumental Lucas paper (1990), which provided evidence and explanation why it is not so, very few scholars have utilized alternate approaches to studying wealth polarization (Berman et al., 2016; Broitman and Czamanski, 2021b, in this volume).

While spatial dynamics are often studied using traditional general equilibrium models (e.g. Zhang et al., 2020), assuming disequilibrium allows for more realistic insights concerning a much richer set of phenomena (Benguigui and Czamanski, 2004; Broitman and Czamanski, 2015).

Indeed, cities and regions are in a constant state of flux (Batty, 2005; Reggiani and Nijkamp, 2006). At times they change very quickly and at other times more slowly. Over an extended period, there are identifiable phase transitions in the evolution of the spatial structure, which often correspond to periods of disruptive industrial and social change. For example, the invention of modern factories in the first industrial revolution transformed villages and small service towns into compact monocentric cities whose outer boundaries were limited by the possible commuting distance. In fact, the mechanization of agriculture and manufacturing in the industrial revolution led to a significant growth of cities. Later, in the early twentieth century, with the invention of beams, nails and mechanized transportation, cities acquired prominent central business districts following the demand-driven steep rent gradients.

The first attempts at studying cities and regions in a state far from equilibrium, without assuming equilibrating forces, appeared some 40 years ago (Batty 1974; 1984). In studies of urban and regional systems, it is now assumed that equilibrium can occur, but its existence is not taken to be imperative.

The heterogeneity of agents in urban and regional systems has various implications for spatial dynamics. While many studies on spatial socioeconomic systems assume that agents are relatively homogeneous, in reality such systems comprise heterogeneous components that self-organize and generate nontrivial morphological structures and behaviors. There are theoretical indications that the extent of heterogeneity affects the system's structure and dynamic behavior significantly, and further that the system's coherent spatial organization is maintained by local interactions, adaptation, information sharing and learning.

Unlike theoretical analyses of such systems, empirical studies are marred by the absence of an agreed-upon definition of the boundaries of various settlement types and the utilization of arbitrary time- and space-dependent measures of various characteristics of the systems to test hypotheses. Data based on different definitions of boundaries and cross-sectional analyses at various stages of systems' evolutions make comparisons of settlement systems and general inferences concerning dynamics problematic. Traditionally, urban models rely on coarse aggregate statistics and relatively small surveys (Glaeser et al., 2018). Big data now provides opportunities but also challenges for studying urban systems (Schintler and Fischer, 2019, 2020; Schintler and Kulkarni, 2014). Because big data has a high degree of resolution and coverage rather than being aggregated and confined to particular spatial and temporal boundaries, it can capture urban variation and heterogeneity in ways not possible with conventional data. Accordingly, big data contributes to our knowledge about urban and regional systems, such as complex mobility patterns (Barabási, 2009; Illenberger et al., 2013; Wang and Barabási, 2021). Big data's main contribution is related to insights concerning variation among objects and groups and not to groups' 'averages'. At the same time, such data are fraught with

imperfections and problems, such as noise, bias and redundancies, which must be managed in their use for urban and regional analysis.

The spatial organization of systems of cities has also received much attention in the past decades. Most of the literature aims to understand the hierarchy of cities within regions, building the work of Zipf (1949), Simon (1955) and Isard (1956). In this context, many discussions have focused on the possibilities and universality of power-law functions – or other functions characterizing regularities in city size distributions (see, among others, Benguigui and Blumenfeld-Lieberthal, 2010; Krugman, 1996a, 1996b; Parr, 1985, 2021, in this volume; Pumain and Moriconi-Ebrard, 1997). As indicated in Section 2.1, the rank–size rule can be derived from entropy maximization, where entropy is linked to spatial interaction and micro-behavioral theory and used as an indicator of the variety or diversity of a system (Reggiani and Nijkamp, 2015). In this context, there is an ongoing debate in the scientific literature about whether Zipf’s law is a statistical regularity or an economic phenomenon (e.g. Berry and Garrison, 1958; Gabaix, 1999; Krugman 1996a, 1996b; Czamanski and Broitman, 2018; Broitman et al., 2020).

Finally, the concept of spatial entropy, derived from Wilson’s work (1967) and later revisited in a geographic population framework (Batty, 2021, in this volume), appears then to play a significant role in the theoretical foundations of spatial dynamics. Thus, there is much promise for entropy to be a fundamental and universal element in an integrated theory regarding the evolution and dynamics of cities and regions and emergent hierarchical structures in such systems.

3. THEMES AND AIMS OF THE HANDBOOK

3.1 A Bird’s-Eye View

Considering the aforementioned reflections and prospects raised, we now discuss the aims and thematic focus of the handbook. To help orient this discussion, we developed a word cloud showing the words most frequently used in the handbook, focusing only on terms that help explain the thematic content of the contributions (Figure 1.1).

The word cloud highlights new and old directions in research on entropy, complexity and spatial dynamics as discussed in the handbook. Entropy and complexity are the major themes, which are indeed woven throughout the handbook in various contexts. Additionally, there is a strong focus on cities and regions, as well as on resilience, networks, economics, space and theory.

The Handbook on Entropy, Complexity and Spatial Dynamics explores new and old theoretical, methodological and empirical insights and perspectives on entropy, complexity and spatial dynamics. A central and cross-cutting theme of the handbook is the role of entropy in complex, dynamic spatial socioeconomic systems as both a measure of spatial complexity and a process involved in the dynamics and emergent behavior of spatial systems. In this regard, one of the bold aims of the handbook is to examine how and to what extent entropy is (or might be) a universal property dictating and describing the structure and dynamics of complex spatial systems across different domains and applications. To this end, the handbook draws upon perspectives from various disciplines – from economics to geography to physics to biology to information science.

Figure 1.1 A bird's-eye view

3.2 Themes and Aims

The handbook summarizes classic and state-of-the-art cross-disciplinary methods and approaches for modeling and characterizing complex, dynamic, integrated networked spatial systems, including entropy-based and information-theoretic frameworks and derivations. It also reflects on and emphasizes how such models and methods can help inform policymaking and planning, particularly for addressing contemporary issues and challenges tied to our increasingly integrated, uncertain and dynamic world.

According to four interrelated themes, the handbook is organized into parts:

- A. Entropy, space and complexity
- B. Complexity of urban evolution
- C. Complexity and resilience of economic systems
- D. Spatial dynamics of complex interactions

Part A provides foundational and emerging insights into the mathematical, conceptual and theoretical linkages between entropy and complexity in spatial and dynamical systems. Additionally, it highlights the range and diversity of entropy concepts and measures across both the social and physical sciences, including novel developments and applications.

Part B focuses on the evolution and complexity of urban areas, including systems of cities. The section highlights how shocks to urban systems give rise to self-organizing behavior and emergent structural properties and regularities, and in this respect how entropic and nonentropic forces and dynamics play crucial roles. It also presents new developments in analytical and computational modeling for characterizing processes and emergent behavior in urban systems.

Part C focuses on the complexity and resilience of dynamic spatial economic systems from theoretical, empirical and methodological perspectives. It also highlights new and old measures and conceptualizations of resilience in integrated and network systems using a cross-disciplinary lens while also considering related methodological and operational challenges.

Part D focuses on spatial interaction and networks, considering and building upon fundamental static and dynamic frameworks, including entropy maximization, discrete choice models and the master equation approach. The section highlights new developments in evolutionary and computational modeling, supported by big data, for simulating how micro-level interaction and gravitational behavior give rise to complex macro-level patterns and properties, such as inequality, heterogeneity and hierarchies. It concludes with some new directions and prospects for developing a general and unifying framework for characterizing and modeling dynamic spatial interaction.

4. *A TOUR D'HORIZON* OF THE HANDBOOK

4.1 From Old Developments to New Insights

Entropy is an outcome and a dynamic in an array of complex dynamic spatial systems across different domains and disciplines. Moreover, entropy and complexity are jointly and intricately involved in spatial systems as sources of discontinuities, nonlinearities and chaotic dynamics (Rosser; all references in this section are to chapters in this volume). Lastly, entropy maximization models contribute to the 'wider complexity agenda' by providing the mathematical foundations upon which 'emergence in complex systems' can be characterized (Wilson).

In this context, entropy concepts and entropy-based frameworks are continuing to move from aggregate to disaggregate, deterministic to stochastic (probabilistic), equilibrium to disequilibrium, static to dynamic considerations and specifications. New developments in the utilization of entropy as an indicator of complexity, such as measuring agglomeration, allow for statistical inference, testing, decomposition and disaggregation across varying spatial and sectoral scales (Cutrini), as well as approaches for working with point-level data (Kopczewska). Models of complex spatial systems increasingly emphasize and reflect entropy as a driving force behind their evolution and emerging structural properties and characteristics. While not always referenced explicitly in terms of entropy, both entropic and nonentropic forces are at play in complex spatial systems, pulling apart and pushing together different elements and features, driving such systems into and out of equilibrium. In urban systems, including systems of cities, certain structural regularities and properties such as Zipf's law and Gibrat's law arise

from such dynamics and processes (including entropy maximization) (Kilgariff et al.; Rybski and Li). But the spatial scale considered – for example, intercity versus intracity – affects the manifestation and precision of such properties (De Montis; Ubaldi et al.). In fact, networks are linked to entropy, as previously indicated and also highlighted by Bianconi: ‘The classical information theory of networks’ on the basis of entropy maximization ‘constitutes a very recent development of the field and provides for the first time an information theory explanation of the origin of the universal properties of networks.’

The resilience of complex spatial systems is a burgeoning area of research attention. Of particular interest is how such systems react and respond to shocks of various types – from pandemics to financial crises – how we can intervene to manage such disruptions, and the role of complexity and entropy in these regards. The need to frame resilience in complexity theory, and further to develop indicators (including those based on entropy) that ‘go beyond equilibrium estimates to acknowledge system dynamism’, is stressed (Haynes et al.). In fact, transformational processes following external shocks in complex systems provide ‘empirical indicators of emergence’ and ultimately resilience (Slaper and Dempwolf). The hierarchical structure of complex spatial systems and ‘relative structural invariance’ of system components concerning one another determine and dictate the capacity for complex systems to react to and recover from shocks – ‘dynamic impulses’ (Scazzieri). The networked nature of spatial systems adds another layer of complexity concerning resilience. Accordingly, modeling is critical for understanding how the functional and structural performance of networks – including integrated systems of networks – respond to various kinds of shocks, and in this regard what can be done to enhance their resilience (Murray).

However, ambiguities and contradictions in the conceptualization of resilience remain, particularly in the context of complex spatial systems. Frankly, there is a lack of a single, unifying definition or measurement of resilience across disciplines, applications and problems (Rose and Dormady). Accordingly, there are conceptual, methodological and operational issues and challenges that complicate our ability to understand and model the resilience of such systems (Batabyal and Kourtiti).

Complex spatial systems are increasingly vulnerable to various problems, including negative externalities, inequality and unanticipated disruptions (Alderighi et al.; Raimbault and Pumain; Sedai and Medda). New developments in the modeling of complex systems emphasize the role of policy and planning in addressing such problems, ultimately to promote broader societal aims of sustainability, resilience and social justice. Towards this end, the complexity of spatial systems can itself be exploited, particularly to help direct and manage transitions such that societal goals and objectives are met (Donaghy). Moreover, the hierarchical and modular nature of complex systems makes it possible to focus on one subsystem at a time while not disturbing the system as a whole (Alderighi et al.; Scazzieri). Entropy has a role to play here, too: ‘entropy provides a lens by which the effects of policy interventions on complex systems can be understood’, especially in systems that are unpredictable, vulnerable to shocks and out of equilibrium (Broitman and Czamanski). In other words, policy instruments can be used to push entropic forces in one way or another to help manage the problems facing cities and regions in our ever-more-complex, dynamic and uncertain world.

Current directions in the modeling and simulation of complex spatial systems involve the use of evolutionary and computational approaches and agent-based frameworks, enabled by new and expanding sources of spatial (and geo-temporal) big data. Such modeling approaches allow us to capture how micro-level interaction and behavior in such systems give rise to

complex structural properties and aggregate dynamics (Hudec et al.; Sedai and Medda). The explosion of spatial big data, driven by the proliferation of location-based devices and platforms, provides new opportunities for characterizing complexity at varying spatial levels of resolution and aggregation (Ubaldi et al.). Moreover, big data, coupled with ever-expanding computing capabilities, enable us to revisit classical theories to test their validity and robustness, particularly considering contemporary trends and transformations. They also allow us to validate studies done before the digital age (Kilgarriff et al.; Östh et al.).

Finally, there is a renewed interest in developing a unifying framework for characterizing and modeling spatial interaction and dynamics in urban and regional systems. In this regard, the dynamic discrete choice model and the master equation approach hold significant promise, with the potential to contribute to further empirical and theoretical advancements in complexity and spatial dynamics (Delloye and Thomas; Haag).

4.2 Lessons and Avenues for Future Research from the Handbook

Considering the new developments and directions in research on entropy, complexity and spatial dynamics highlighted in this brief synthesis, we now highlight some lessons and suggestions for further research.

'Beg, borrow and steal', with caution. Exploiting theories, concepts, models and methods from diverse disciplinary perspectives, including those outside the field of spatial economics, can significantly improve our ability to understand and model complex spatial systems (e.g. Bonin; Dentinho and Rodriguez; De Montis; Raimbault and Pumain; Tellier). However, while spatial socioeconomic systems and physical systems, including ecological systems, share common properties and dynamics, they are also nuanced and distinct in various regards. Accordingly, when transferring and translating concepts from one discipline and application to another, we need to be able to delineate between 'true' and 'superficial' analogies. Towards this, we need to fulfill various methodological conditions and make appropriate applications given the system under consideration. For example, we should 'distinguish statistical entropy/information from thermodynamic entropy', using references from physics only when it makes sense to do so, such as for 'ecological or nature–human interface studies' (Haynes et al.).

Context matters. For complex spatial socioeconomic systems, consideration of context is critical for anticipating possible trajectories under a range of scenarios, including different kinds of shocks (Scazzieri). Indeed, efforts to evaluate and plan for complex spatial systems (including spatial networks) occur in varying contexts, and an understanding of contextual factors and dynamics is imperative for designing robust and resilient systems (Murray; Rose and Dormady). Accordingly, context may help in disentangling ambiguities and asymmetries in conceptualizations of resilience, and of entropy and other elements of spatial complexity, for that matter.

'Location, location, location.' In portraying and modeling complex spatial systems, the geographic dimension must be treated as a unique attribute and dynamic. Indeed, space itself in socioeconomic systems plays a role in dictating complex processes in and outcomes of systems (Rosser). Thus, further research is needed to understand how geography plays an explicit and implicit hand in polyplexity, the evolution and structure of complex spatial systems – such as the dynamics and distributional properties of systems of cities (Parr). In fact, geolocated data itself may be a proxy for complex spatial dynamics, such as those involving human mobility (Ubaldi et al.).

Occam's razor. Complex spatial socioeconomic systems are 'supercomplex' phenomena, composed of interrelated social, economic and physical systems operating at different spatial and temporal levels and hierarchies (Alderighi et al.; Scazzieri). To capture such complexities, we should consider mixing and integrating theories and models across different disciplines and domains (Berry; Tellier). However, in doing so, including 'more features and dynamics' in models of complex systems not only adds to computational complexity but also muddies our ability to understand the systems (Hudec et al.). Therefore, we need techniques and approaches for addressing these issues and challenges (Sedai and Medda).

Big data as a big solution with some big caveats. New and expanding sources of data – big data – along with related computational tools and methods have the potential to enhance our ability to model and conceptualize complex spatial systems. From a technical perspective, we need to develop robust approaches for managing data in different formats, such as point patterns, and in this regard to move from disaggregate to aggregate levels in a consistent manner (Kopczewska). There are also basic data-collection needs to ensure that we have observations and data points for the 'many analytical levels and scales within a complex system' (Slaper), and big data may help with this. We should take the era of big data as an opportunity to validate, question and refine classical theories and laws (Kilgariff et al.; Östh et al.; Rybski and Li) and develop 'new perspectives on measurement' of spatial complexity, especially given that spatial systems become increasingly complex as they 'grow and evolve' (Batty). However, it is vital to recognize the shortcomings of big data. Their volume, velocity, variety and variability call for analytics to improve veracity and value for urban modeling.

Statistical and probabilistic foundations. It is important to explore statistical and probabilistic foundations and constructs of spatial complexity, including entropy (Haynes et al.; Kopczewska). Along these lines, in our development of indicators of spatial complexity, including entropy-based, we need to have methods for statistical testing and inference while also ensuring that there is a meaningful null hypothesis (Cutrini). Additionally, traditional statistical approaches such as 'indicators of spatial dependence and spatial econometrics' may be needed to understand the geographic nuances and properties of complex socioeconomic systems, particularly in systems that 'exhibit interdependent linkages among variables in adjacent regions' (Batabyal and Kourtit).

Disequilibrium, discontinuities and dynamics. Many of the measures of spatial complexity – including those that are entropy-based – are based on static frameworks and considerations. Accordingly, there is a need to develop concepts and indicators that reflect 'system dynamism' and go beyond assumptions of equilibrium (Broitman and Czamanski; Donaghy; Haynes et al.).

The omnipresence of entropy. Entropy is at play in all complex spatial phenomena as an outcome and a dynamic, as centrifugal and centripetal forces (Wilson). The particular role of entropy and its conceptualization across different types of complex system is less known. Moreover, there is a need to develop a 'dynamics of entropy' that goes beyond static formulations from statistical mechanics (Batty; Haynes et al.) and to develop applications of entropy-based aggregation and disaggregation across different spatial scales and sectors as well as networks (Batty; Bianconi; Cutrini). Additionally, more research is needed to explore the extent to which the dynamic discrete choice model is compatible with the master equation approach and the maximization of a dynamic entropy function (Delloye and Thomas; Haag).

Returning to the possibility of entropy as a unifying principle that enables us to bridge probabilistic approaches with behavioral approaches, new horizons and research trajectories

highlighted in this handbook help move us in that direction. However, in light of new theoretical advances, there is still much more to be done, necessary for paving the way towards even more fruitful applications. We need to make sure that frameworks are logically and mathematically consistent, and to this end cross-disciplinary perspectives are needed. Indeed, this is a grand and exciting avenue for further research. If successful, could such efforts and developments contribute to a rebirth of theory?

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