Principles of capitalistic commodity production reconsidered

Jens Reich*
Department of Economics, Johann Wolfgang Goethe Universität, Frankfurt am Main, Germany

Helmedag (2012) derives effects on employment caused by changes in (a) uncompensated wage hours, (b) output, (c) productivity, and (d) a combination of the latter two. His results are derived from a linear two-sector model. His closure of the model is based on the determination of the profit rate via aggregate production. This closure bears some flexibility. Changes in the profit rate require changes in the composition of output, but the level of total production cannot be derived uniquely thereof. It will be shown, therefore, that results (a), (c) and (d) depend on a specific assumption. Without it, different adjustment paths are possible. The model is then either limited to economies of certain institutional characteristics, or a plea for certain institutional elements, such as social insurance systems.

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1 RECAPITULATING THE MODEL

Helmedag pictures a closed economy without a government, but he argues that his results could be extended to such cases with no qualitative changes (see Helmedag 2012: 23). In his approach to economics, he combines the labour theory of value with a modern formulation of the classical approach to economics. Following Marx’s suggested solution of the transformation problem, Helmedag aggregates the economy in labour values. This aggregate is then split into two analytically constructed subsystems. The first is the wage sector, accounting for all goods which are purchased from wages. The second is the luxury sector, accounting for all goods purchased from profits. The subsystems are expressed in a fashion which reminds the reader of Sraffa’s (1960) Production of Commodities by Means of Commodities. The two sectors are characterized by two linear equations, relating the profits and labour cost to the net product. Helmedag’s approach is based on labour values and should therefore rather be characterized as a ‘Production of Commodities by Means of Labour’. This article is a comment on the model resting upon this basis. It is not a treatment of the general underlying economic framework.

Equation (1) represents the wage–good sector. Equation (2) represents the luxury good sector. The equations used here are reformulations of Helmedag’s equations (H-6a) and (H-8).

\[(1 + s) L_B w_B = B p_B, \text{ where } B v_B = L_B. \] (1)

\[s = \frac{B p_B - B v_B w_B}{B v_B w_B}. \] (H-6a)

\[s = \frac{X p_X - X v_X w_B}{X v_X w_B}. \] (H-8)

* jreich@wiwi.uni-frankfurt.de.

1. (H-6a): \(s = \frac{B p_B - B v_B w_B}{B v_B w_B}. \)
2. (H-8): \(s = \frac{X p_X - X v_X w_B}{X v_X w_B}. \)
The equations are read in the following manner. The production of basic goods \((B)\) requires an amount of working hours \((L_B)\). The relation of output and labour input is defined by the productivity \((v_B)\). Working hours have to be compensated by the wage rate \((w)\) and profits are paid as a markup on wage cost \((s)\). The luxury sector is of the same structure. Thus, Helmedag’s model is a linear two-sector model, consisting of four unknowns and two equations. Due to the simple structure, prices are determined by the relative productivities of the two sectors. We get Equation (3) by dividing Equations (1) and (2).

\[
\frac{p_X}{p_B} = \frac{v_X}{v_B}.
\]

This is identical to Helmedag’s equation (H-9), which represents a special case. Relative prices are independent of distribution. This avoids the transformation problem. The relative price of luxury goods in terms of basic goods is the only relative price in this system and it may therefore simply be called ‘\(p\)’. This is analogue to Helmedag (see equation (H-8)) and corresponds to choosing the price of wage-goods as numeraire. Equation (3) can be rewritten as:

\[
p = \bar{p}_X = \frac{p_X}{p_B} = \frac{v_X}{v_B}, \quad \text{with} \quad \bar{p}_B = \frac{p_B}{p_B} = 1.
\]

Choosing a numeraire reduces a degree of freedom. The so far undefined distribution is characterized by an antagonistic relation of profits and wages. For a constant productivity, either of them can only increase its share of output if the other’s is reduced. Only if productivity changes can the wage rate and the profit rate rise or fall simultaneously. This relation can be expressed as a linear relationship by dividing Equation (1) by the value of output \((p_B B)\).

\[
(1 + s) w v_B = 1.
\]

This highlights that the distribution is determined in the wage–good sector. This is equivalent to Helmedag’s equations (6b) or (6c).\(^3\) We have a system of two equations, (1) and (2), their reformulation (4) and (5), and three unknowns \((p, w, s)\). For a given labour productivity, one degree of freedom is left and expressed by Equation (5). Either the profit rate or the wage rate have to be determined to solve the model.

2 SOLVING THE MODEL

Helmedag uses a macroeconomic or ‘Keynesian’ closure of the model. According to his definition of wage and luxury goods it follows that the total wage bill is spent on basic goods (expressed by Equation (6)) and that profits are spent on luxury goods (as expressed by Equation (7)). This is a variant of the well-known ‘Keynesian’ assumption about the behaviour of workers and entrepreneurs. It is usually paraphrased as: workers

3. (H-6b): \(s = \frac{1}{w} v_B\) or (H-6c) \(s = \frac{1}{v_B} w\).
spend what they earn and capitalists earn what they spend. This assumption provides a further constraint to the system.

\[ w(L_X + L_B) = B, \text{ and} \]

\[ s w(L_X + L_B) = Xp. \]

These equations express the necessary condition such that wages are high enough to consume all wage goods and that profits suffice to buy all luxury goods. Within this ‘pigovian’ approach it has to be true, because wage goods were defined as the goods purchased from wages whereas profits are spent on luxuries. Using Equations (6) and (7) this can can be summarized in one equation:

\[ sB = Xp. \]

If we now replace the output of wage goods and luxuries (B and X) using the definition of productivity (see Equations (1) and (2)) the relative productivities and relative prices cancel out following Equation (4). What remains are the labour inputs to both sectors, which lead to:

\[ s = \frac{X}{B} p = \frac{L_X}{L_B}, \text{ or } sL_B = L_X. \]

This is equivalent to Helmedag’s equation (H-10). Every distribution fits with a certain composition of output and input. If the composition of input or output changes, distribution has to change, and vice versa. The microeconomic distribution (the wage and profit rate) and the macro-level have to adjust to each other. In Helmedag’s model the distribution determines the composition of output. If distribution changes, the composition of output has to change, too. Alternative regimes of growth, income distribution, and the composition of output, two being fixed and the third one determined endogenously, are treated in a similar fashion in Walsh/Gram (1980) or Schefold (1997).

3 DISTRIBUTION AND THE LEVEL OF EMPLOYMENT

It was shown above that the distribution is determined by the composition of output. For given levels of labour inputs, the composition of output is determined. This ratio determines a certain profit rate. Helmedag reverts this causality to derive his results. He asks for the implications of changes, for example in the microeconomic distribution on the level of employment. Total employment can be expressed as the sum of the labour inputs of both sectors (L). From Equation (9), we know that the labour input of the luxury sector (L_X) equals s times the labour input of the wage–good sector (sL_B). It follows that total labour demand is proportional to either of the labour inputs: (L_B + sL_B) for the wage good sector and (1/s L_X + L_X) for the luxury sector. Using the definition of labour productivity, we can replace the labour input by the product of productivity and output (v_BB or v_XX). Doing this, we get Equation (10). Total labour

\[ \text{(H-10): } L_B = \frac{L_X v_B}{v_B - v_{GW}} = \frac{L_X}{s}. \]
demand depends on the level of output/production, distribution, and productivity. This has to be true for the wage–good sector as well as for the luxury sector.

\[ L = (1 + s)v_B = (1 + \frac{1}{s})v_X X. \]  

(10)

Helmedag’s corresponding equation is (H-11).\(^5\) This equation is used to study the implications of changes in the micro-distribution on the absolute level of employment. However, the relation of the absolute levels of labour inputs and the distribution is not biunique. Certain absolute levels of labour inputs uniquely determine a rate of profit, but a given rate of profit solely determines a unique \textit{composition} of labour inputs, not their absolute levels. The absolute levels cannot therefore be inferred from the distribution. Alterations of the profit rate or productivity may result in a variety of new absolute levels of employment. Without further assumptions it is unclear which level of total employment will result. This is obvious in Equation (9), but it can also be highlighted in Equation (10). To do so, the equation is restated in growth terms.\(^6\) The subscripts indicate the variable.

\[ g_L = g_s + g_{vb} + g_B = g_s' + g_{vx} + g_X. \]  

(11)

It is important to remember that productivity was defined such that a rise in productivity requires its variable (\( v \)) to shrink \((g_{vb} < 0)\). Additionally, the profit rate of the luxury sector enters in the denominator. A rise in the profit rate \((s)\) corresponds to a decline of its reciprocal \((g_s' < 0)\). This is just a different way of expressing what was already stated in Equation (9). The modelled economy is constrained by the composition of output, distribution and productivity. A change in either of the variables (total working hours \((g_L)\), distribution \((g_s' \text{ and } g_s)\), productivity \((g_{vx} \text{ or } g_{vb})\), or the composition of output \((g_X \text{ or } g_B)\)) has to be matched by a change in at least one of the other variables. The possible combinations span a space limited by Equation (11).

4 \hspace{1em} \textbf{THE RESULTS}

Helmedag derives his results regarding the total level of employment from equations (H-6a – c), (H-10) and (H-11). These are equivalent to my Equations (5), (9) and (10) (or if preferred (11)). He argues to have proven that: (a) ‘working longer without appropriate real wage adjustments raises the rate of profit and hence causes dismissals’ (b) ‘a falling production of luxuries diminishes employment not only in this branch but in the basic sector too’, (c) ‘labour productivity growth in the wage–good sector accompanied by an under-proportional increase in pay leads to lay-offs’, and (d) ‘even if productivity improvements in the basic sector are matched exactly by higher real wages, the workforce shrinks whenever output per capita in the luxury division also expands’ (Helmedag 2012: 31). Thus, Helmedag derives results from changes in distribution, productivity, and the level of production of luxuries regarding the total level of

5. \( (H-11): L = L_X + L_B = \frac{L_s}{1 - v_{wb}}. \)

6. Growth rates can be derived if we take the logarithm and differentiate with respect to time: \( g_z = d\ln(z)/dt \). If time is understood as ‘theoretical’ rather then ‘historic’ time, the equation simply restates the equilibrium requirements of Equation (10). It expresses the possible equilibrating changes if either variable would be changed. It is therefore free of causality.
employment. However, it was shown above that a certain distribution fits with different levels of employment. The reason is that a certain distribution fits with different total levels of employment, if these levels are characterized by the same composition of labour inputs. It seems questionable whether such results can be obtained unambiguously. Let us look at his results one by one.

Helmedag’s first result is that (a) an extension of working hours at the firm level, which is not compensated by higher wages, leads to a rise in the rate of profits \( g_s < 0 \). Let us trace the argument. The extension of working hours alters the distribution of the economy. The rate of profit rises and the wage rate falls. This change in distribution requires a new composition of labour inputs (and output). From Equation (9), we know that, for a rise in the profit rate, more workers have to be employed in the luxury sector. The suggested path to adapt to this change is a reduction in total working hours \((g_L < 0)\). Thus, the rise in the profit rate is compensated by a fall in employment. The argument can be summarized as: (a: \( g_L = g_s < 0 \), where \( g_{vb} = g_X = 0 \)).

There are, however, alternative paths which also satisfy Equations (5), (9) and (10) (or (11)). The change in the composition of labour input may also be reached by an expansion of the luxury sector. Let us sketch the range of possibilities in Helmedag’s framework. If the higher profit rate \( g_s < 0 \) leads to a higher demand for luxuries \( g_X > 0 \), the effects on employment may be negligible or reversed. If the increased demand for luxuries exactly matches the fall in demand for wage–goods, total employment may remain constant \((g_L = 0)\). This can be summarized as (a’1: \( g_{L} = g_{s} + g_{X} = 0 \), where \( g_{X} > 0 > g_{s} \)). The shift in demand may even increase total employment if the changed demand structure requires more employment to be satisfied. In this case, the wage–good sector expands as well \((g_{B} \geq 0)\) and total work hours rise \((g_{L} > 0)\). The rising profit rate induces a growth in both sectors \((a'2: g_{L} = g_{s} + g_{X} > 0 \), where \( g_{B} \geq 0 \)). All three paths (a, a’1, and a’2) satisfy all equations. The labour inputs and outputs adjust such that their relative shares fit to the new distribution.

The second result is that (b) a reduction in the luxury production \((g_X < 0)\) leads to layoffs in both sectors \((g_{L} < 0)\). Let us again trace the argument. The proportion of both sectors is determined by the distribution. If the production of luxuries is assumed to be reduced, the wage–good sector has to shrink to adjust its relative size if distribution and productivity are held constant.

Helmedag’s third result is that (c) a rise in labour productivity in the wage–good sector \((g_{vb} < 0)\), which is not fully compensated by wage increases leads to a reduction in employment \((g_{L} < 0)\). Let us trace the argument. A rise in productivity in the wage–good sector \((g_{vb} < 0)\), which is not fully compensated by a rising wage rate, requires that the rate of profit rises \((g_s > 0)\). As the wage rate rises, the growth in the profit rate is, according to Equation (5), smaller than the rise in productivity \((g_{vb} + g_{s} < 0)\). The increased productivity in the production of wage–goods increases the relative price of luxuries. Thus, again, the luxury sector has to expand relative to the labour–good sector. Helmedag’s suggested path is a contraction of labour–good production with no change in the demand for luxuries \((c: g_{L} = g_{vb} + g_{s} < 0 \), where \( g_{X} = 0 \)).

Alternatively, the economy can adapt by an expansion of the labour–good \((g_{B} > 0)\) or by an expansion of the luxury–good demand \((g_{X} > 0)\). The increased productivity can be used to expand the supply of wage–goods to the extent that the wage rate was increased, thus the productivity growth minus the growth in the profit rate \((g_{B} \geq g_{vb} - g_{s})\). Due to the change in the profit rate \((g_{s} > 0)\), the increased productivity cannot completely be offset by the increased production of wage–goods. The composition of labour inputs has to change as well. Equation (5) can be fulfilled either by changing the composition of labour input, leaving total employment constant, or by
a total rise in employment. In the first case, the expansion of the wage–good sector exactly matches the gap between the change in productivity and distribution. Thus, the lay-offs in the wage–good sector are matched by new employments in the luxury production (c’1: \( g_L = -g_v - g_s + g_B = 0 \), where \( g_B = g_v - g_s \)). In the latter case, the expansion in the production of wage–goods overcompensates the increased productivity and profit rate (\( g_B > g_v - g_s \)), such that total employment rises (c’2: \( g_L = -g_v - g_s + g_B > 0 \)). All solutions (c, c’1, and c’2) satisfy the assumptions and all equations. Thus, the paths are all possible within the sketched framework.

The fourth and last result (d) is that an increase in productivity in the wage–good sector (\( g_v < 0 \)), which is accompanied by a proportional increase in the wage rate (\( g_v < g_s = 0 \)), leads to unemployment (\( g_L < 0 \)), if output per capita in the luxury sector is increased (\( g_s + g_v < 0 \)). Let us trace the argument. A rising productivity (\( g_v < 0 \)), which is compensated by a proportional rise in the wage rate, leaves (following Equation (5)) the profit rate untouched (\( g_s = g_s = 0 \)). The increased productivity can be used to cut back on inputs, which would induce employment to shrink (\( g_L = g_v < 0 \)) or the gain in productivity could be used to increase the supply of wage–goods (\( g_B > 0 \)), such that the increased productivity is compensated or even overcompensated by a growth in production (\( g_v + g_B > 0 \)). Both paths fulfil the characteristic equations. If we add the assumption that the production of luxuries expands per capita (\( g_x + g_v = g_v < 0 \)), both paths are still possible. In cases where the demand for luxuries remains constant (\( g_x = 0 \)), employment has to fall. The argument can be summarized as: (d: \( g_L = g_v + g_B = 0 \)). Both paths fulfill the characteristic equations. If we add the assumption that the production of luxuries expands per capita (\( g_x + g_v = g_v < 0 \)), both paths are still possible. In cases where the demand for luxuries remains constant (\( g_x = 0 \)), employment has to fall. The alternative paths (a’, c’, and d’) required a change in the ‘consumption’ of luxuries (\( g_x > 0 \)). The ‘simplifying’ assumption is therefore of the highest importance and is a necessary requirement to achieve the results obtained. Furthermore, the luxury is not only autonomous but is necessary to be fixed. The difference is important. Keynes assumed investment to be autonomous, but he clearly never intended it to be constant. To Helmedag’s

7. The latter could also be understood as a rise of luxury production per population. In this case, total output of luxuries has to grow faster than the population. For a zero population growth, it requires a growth in luxuries (\( g_x > 0 \)). A fall in employment (\( g_L < 0 \)) leads in this case to an immediate contradiction between Equations (10) and (11), (\( g_L \neq g_x \)). Thus, the ‘output per capita’ has to be understood as the output per workforce.

8. The ratio of labour inputs and the ratio of output were shown to be proportional. A change in the productivity of one of the sectors appears to alter this proportion. The output of wage–goods relative to luxuries has to rise due to the increased productivity if the relation of labour inputs remains constant, as required by Equation (9). However, the change in productivity alters the relative price of luxuries (\( p \)). The rising number of wage–goods is compensated by a rising value of luxuries such that the relation of labour inputs and output remains proportional.
results it is of less importance that the demand for luxuries is autonomous, but it is of the highest importance that the demand for luxuries remains constant. The achieved results fully depend on this assumption, and the results are therefore not robust. Slight variations in the assumption change the outcome of the model. Due to its importance, the assumption should not have been introduced within a thought experiment, as it appears to be limited to the latter, and it should not have been called simplifying but instead the importance of a fixed level of luxury demand should have been acknowledged.

5 INTERPRETATION

Due to its importance, it is worthwhile to interpret the simplifying assumption. An absolute limit to the demand for luxuries requires that those who live off profits are fully saturated by this absolute level. A rising rate of profit could easily induce rising total profits and a rising quantity of luxuries, because any purchase financed out of profits is defined as a luxury. What the assumption expresses is that the recipients of profits have no incentive to increase their absolute level of expenditure. They are fully saturated. An incentive to invest and produce is excluded by assumption. It seems questionable whether such an assumption is applicable in a world of big central governments and financial markets. Lending via financial markets might serve as an almost unlimited channel for increased purchases from profits, and government purchases (which would have to be included in the autonomous ‘luxury’ demand) render a fixed absolute level of autonomous luxury consumption implausible. The real cases to which this model applies are therefore rather limited. To extend its application it may be asked whether its results may hold true without the simplifying assumption. Taking the underlying production into account it is possible to identify ‘the driving forces beneath the surface’. In all analysed cases, an exogenous shock distorted the previous equilibrium. The economy faces an ‘overproduction’ of wage–goods and a ‘shortage’ of luxury–goods. Either caused by a change in the profit rate (a), productivity (c), or a combination of both (d), a change in the composition of labour inputs and output became necessary. With a flexible supply and demand for luxuries the economy may adapt via an expansion or a contraction. Instead of a shrinking wage–good sector, the economy might alternatively reach an equilibrium at a higher level of total production, as shown above.

Let’s first consider a special case. If the wage–good and the luxury–good are identical in their qualitative structure it is most likely that changes in distribution leave the level of employment untouched. What is deducted from the wage bill is redistributed to profits. As the same goods are demanded, the underlying production does not have to adapt. Former wage–goods simply become luxuries as they are now purchased from profits. Negative effects on employment are unlikely, except for the additional assumption of an upper level of luxury demand.

If the luxury and the wage–good are qualitatively distinct, changes in productivity or distribution require the underlying production structure to adapt. A shift from wages to profits causes relative prices to change either temporarily, in the case of changes in distribution, or permanently, in the case of changes in productivity. Those industries supplying the commodities which mostly enter in the luxury–good have an incentive to expand and the industries supplying what is mainly accounted as a wage–good have the incentive to shrink. The impact on total employment depends on the strength of the expansionary and contractionary forces. The expansionary forces depend on the adjustment speed of those industries facing a rise in demand. The stronger and faster
the reaction in their supply, the stronger the expansionary effect will be. The adjustment speed most likely depends on the degree of competition for market shares amongst the competitors. The stronger the competition, the more likely is an expansion of employment and production in the expanding industries. The lower the competition, the smaller the expansionary effect will be. The contractionary forces depend on the adjustment of those industries which face a depressed demand. They can adapt faster the more flexible the labour market. The more rigid the labour market, the more time it takes to cut back on employment. The stronger lay-offs are delayed or compensated, for example by labour market regulation or by unemployment benefits, the smaller the contractionary effect. Summarizing, it can be said that economies characterized by highly competitive goods markets and well regulated labour markets are more likely to adapt via expansions, while economies characterised by a low level of entrepreneurial competition and highly flexible labour markets are more likely to adapt by a contraction. Thus, without the assumption of an exogenously determined demand for luxuries, Helmedag’s results still apply in a world without a strong government, without social insurances, and unregulated labour markets. The model can therefore be understood as a plea for a strong public sector and social insurances.

REFERENCES